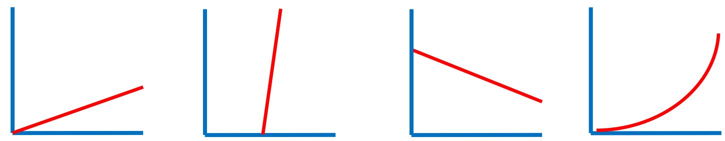
Lesson Title: Date:

(A) Lesson Objectives:

a. Investigate multiple representations of the inverses of linear and exponential functions
b. Graph inverses of exponential functions
c. Explore real-world applications of inverses of linear functions

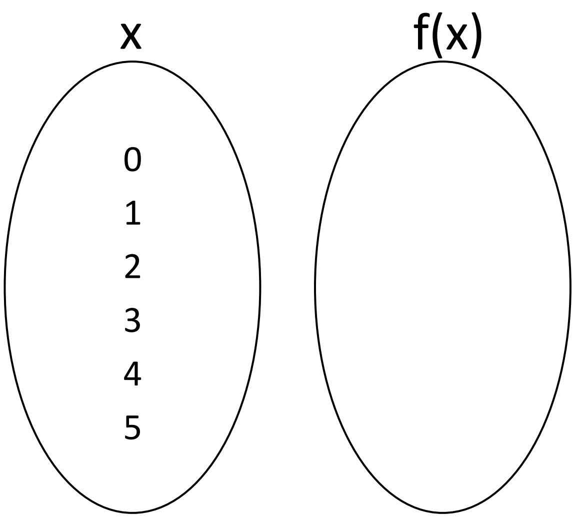
(B) Fast Five:

For each of these graphs, consider what would happen if you swapped the x and y coordinate of each point on the line. Draw a new line to show your prediction.



(C) Explorations:

Complete this mapping diagram to show the function f(x) = 2x – 1 for the given domain.

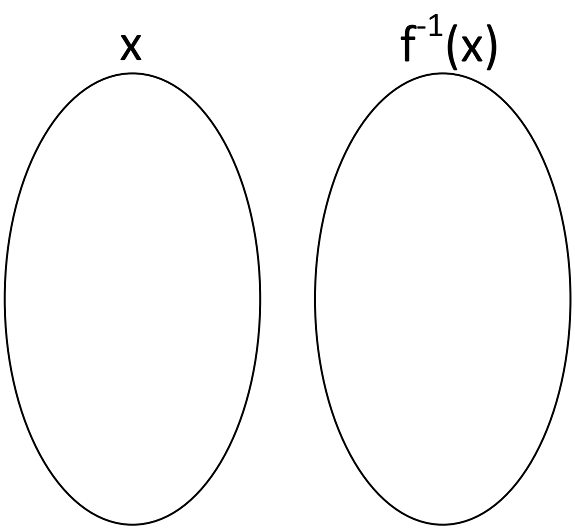


Concept questions:

Find f(3): ____________________

Solve f(x) = 7: ____________________

Now complete the next mapping diagram to show the inverse of the function. You shouldn't need to do any algebra at this point.



Concept questions:

Find $f^{-1}(7)$: ____________________

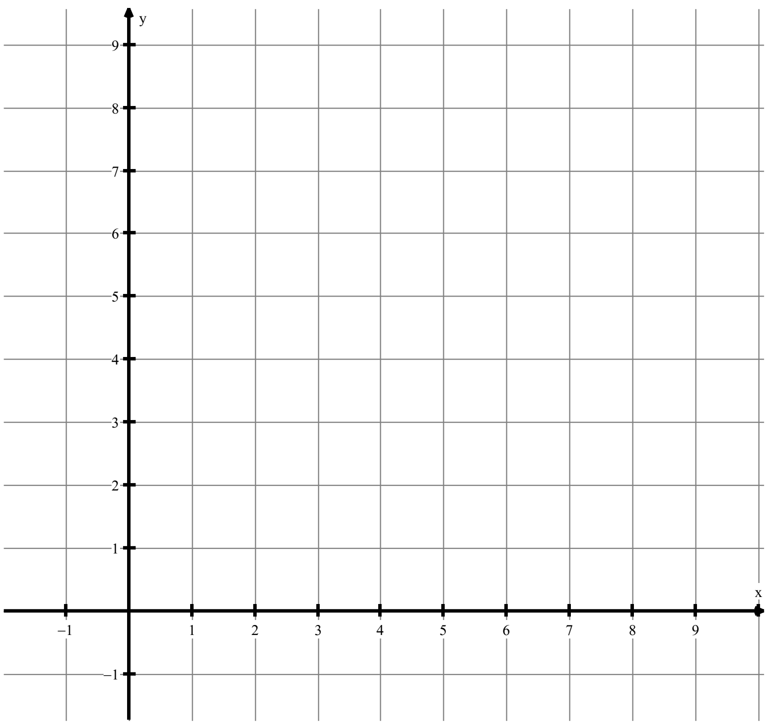
Solve $f^{-1}(x) = 3$: ____________________

(D) Graphing Inverses

Construct a data table for the function **y = f(x)** where **f(x) = 2x – 1**.

x	0	1	2	3	4	5
y						

Draw the graph of **y = 2x – 1 (0 < x ≤ 5, x ϵ ℝ)**



Using your data table from (C), **plot the inverse** function on the same graph. What do you notice?

(E) Swapping x and y

- On the same graph, now plot the line y = x. Remember to label all three of your lines.
- Draw a dotted line between (2, 3) and (3, 2). Notice that the function and its inverse are **reflections** of each other **in the line y = x.** What is the angle between your dotted line and the line y = x? ________________
- Find two other pairs of corresponding coordinates and draw dotted lines between them. Note down the coordinates here:

First pair: ________________________________ **Second pair:** ________________________________

What do you notice about the x and y values of these corresponding points?

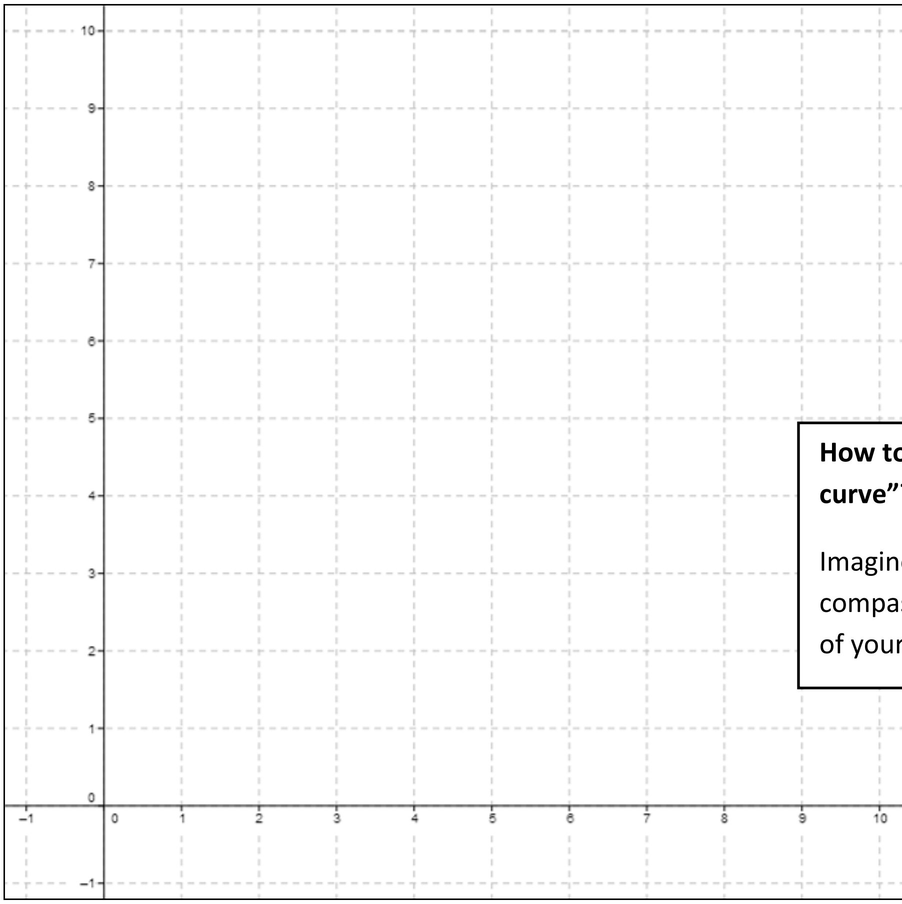
____

(F) Use your GDC to construct a data table for the function **y = f(x)** where **f(x) = 1.4^x**.

Round values to 1 dp

x	0	1	2	3	4	5	6	7
y								

Plot the graph of **y = f(x)**. Draw a **smooth curve** between the points.



How to draw a "smooth curve"?

Imagine your hand is a compass and the "heel" of your hand is the point.

(G) Now consider the **inverse** of the function f(x) = 1.4^x. **Predict** what its curve will look like on the graph and **test** your prediction by listing 3 sets of coordinates that you think lie on the curve of $f^{-1}(x)$.

First: ________________ **Second:** ________________ **Third:** ________________

Without using your GDC or doing any calculations, **fill in** this data table for $f^{-1}(x)$. *(Hint: How is this data table different from others you've filled in? Why is it like that?).* **Plot** the curve on the graph.

x								
y	0	1	2	3	4	5	6	7

(H) Inverses of exponential functions

You can describe functions in words.

f(x) = 2x – 1 can be described like this: "You input a number x and the output is twice x, minus one."
f(x) = 2^x can be described like this: "You input a number x and the output is two raised to the exponent of x."

Think about the inverse of f(x) = 2^x. Describe the **INVERSE** function in the same way:

____

(I) Applications of inverses of exponential functions

What is the **general form of an exponential function**? y = ______________

When we calculate the output of an exponential function, we know **a**, we know **b** and we know **x**.
We *calculate* **y**. This helps us answer questions like this:

In 2012, Mr Toze puts $250,000 in his bank account, which pays him interest of 4%.
How much will he have at the end of 2020?

Check that you know what a, b and x are by describing them and stating the value each has:

a = Description: ____
Value: ______________

b = Description: ____
Value: ______________

x = Description: ____
Value: ______________

Now **describe** and **calculate** y:

y = Description: ____
Value: ______________

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When we calculate the output of the **inverse** of an exponential function, we know **a**, we know **b** and we know **y**. We *calculate* x!

See if you can **rephrase** the question about Mr Toze's finances so that it is now x that we are trying to find:

____

____

____

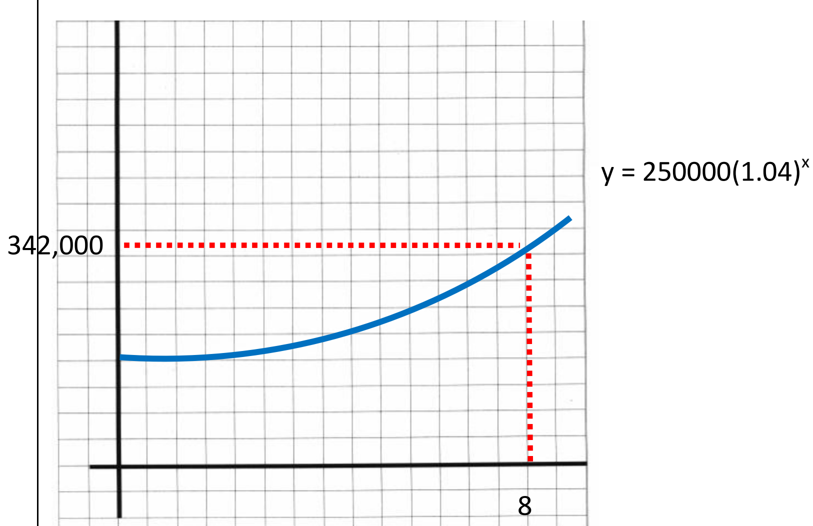
These types of questions are tackled algebraically in IB Mathematics SL using **logarithms**, which are the name given to the inverses of exponential functions. Until then, you can solve these equations graphically by plotting the function, drawing a smooth curve, ruling a line from the curve to the y-axis at the correct point and then reading the corresponding value from the x-axis.

Question:

Mr Toze starts with \$250 000, which he invests at an annual growth rate of 4%. He ends up with \$340 000 (2 sf). How long did it take him to make this much money?

Variable	Description	Value
a	The amount Mr Toze started with	250000
b	The annual growth rate	1.04
x	The number of years	Unknown
y	The amount Mr Toze ends with	342000

Graphical solution



x	y
0	250000
2	270400
4	292500
6	316300
8	342100

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Question:

Graph the inverse of the function above by switching x and y in the expression

$y = 250000(1.04)^x$

to give

$x = 250000(1.04)^y$

Variable	Description	Value
a	The amount Mr Toze started with	250000
b	The annual growth rate	1.04
y	The number of years	Unknown
x	The amount Mr Toze ends with	342000

Graphical solution

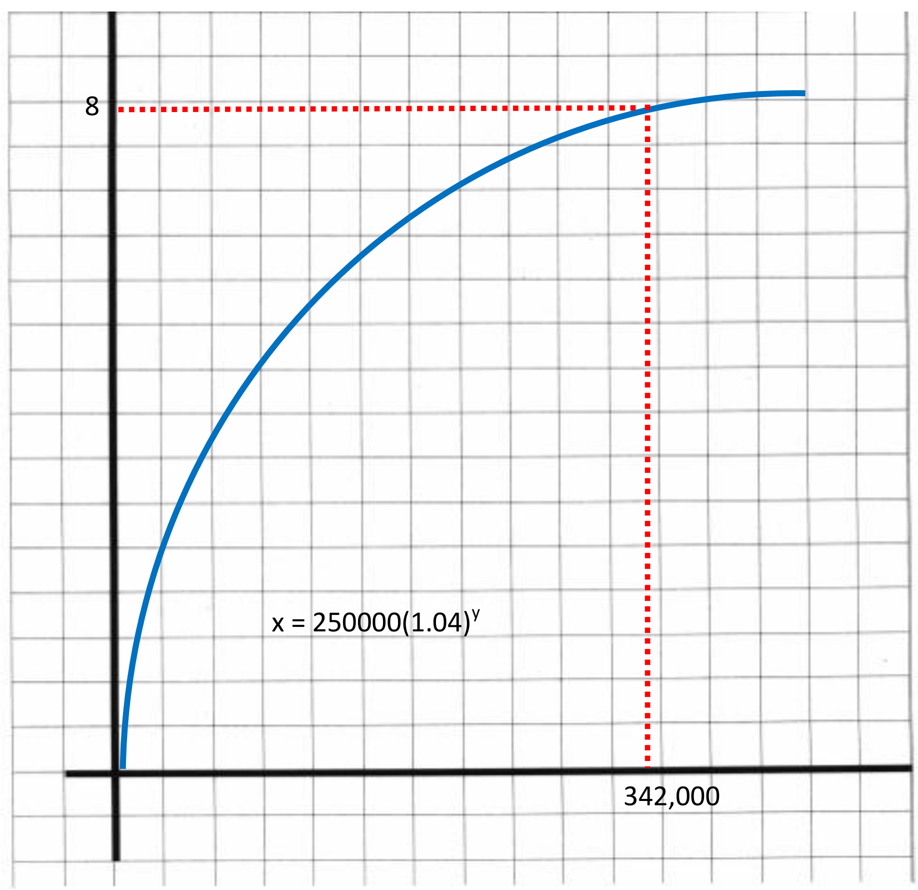

Table of values

x	y

Lesson Title: Date:

Question:

If $x = 250000(1.04)^y$, then:

$1.04^y = x/250000$

Graph the following function using your GDC.

$y = \log_{1.04}(x/250000)$

What do you notice? What is the relationship between the log function and the exponential function?

Variable	Description	Value
a	The amount Mr Toze started with	250000
b	The annual growth rate	1.04
y	The number of years	Unknown
x	The amount Mr Toze ends with	342000

Graphical solution

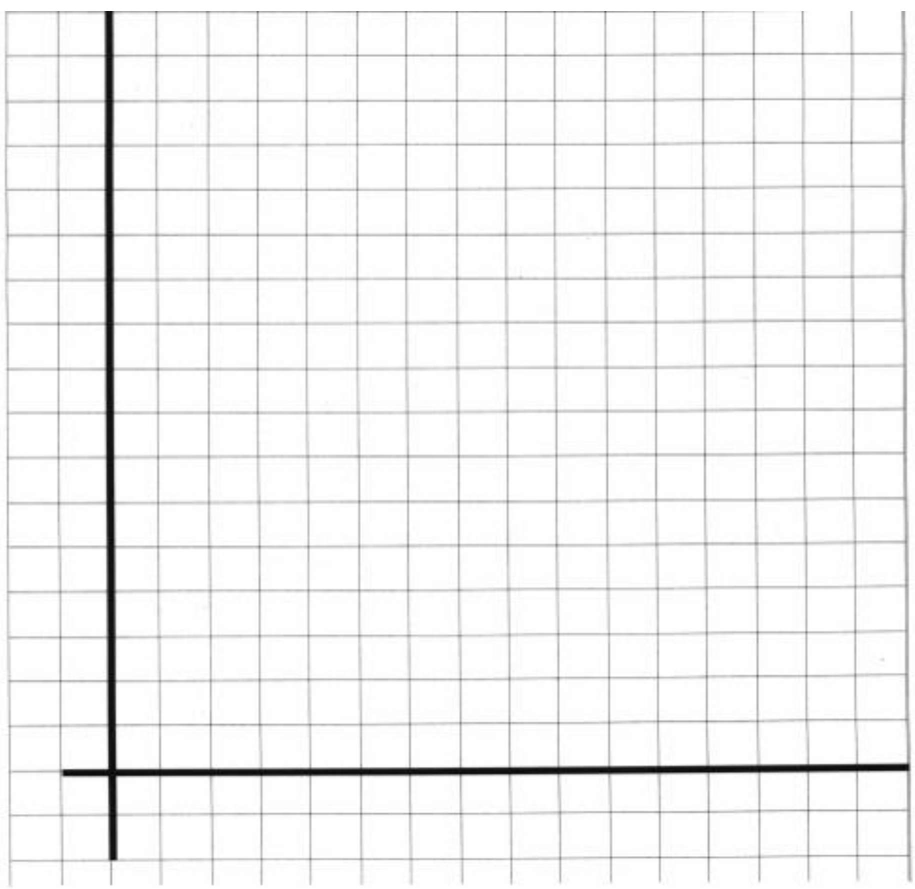

Table of values

x	y

(J) Graphical solutions to problems involving inverses of exponential functions.

Think up your own problems involving inverses of exponential functions. All you need is:

- a story line
- a value for a
- a value for b
- a value for y

Plot a graph to model the problem and use a vertical dotted line from the curve to the x-axis and a horizontal dotted line from the curve to the y-axis to show the solution. Don't forget to label the diagram with:

- Scales of the axes
- The equation of the curve
- The x and y values at the solution

Question:

[Construct your own question by choosing values for a, b and y.]

Variable	**Description**	**Value**
a		
b		
x		Unknown
y		

Graphical solution

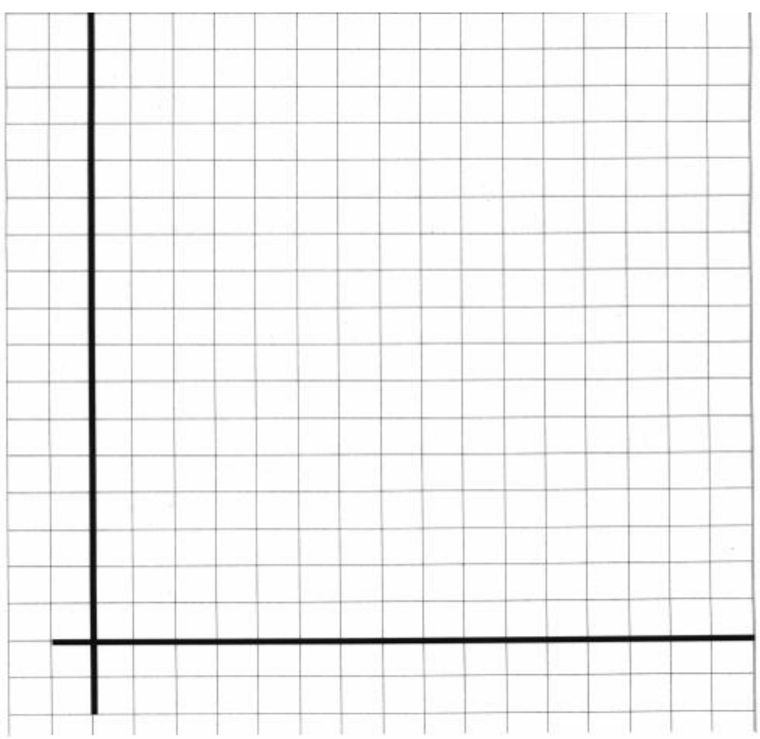

Table of values	
x	**y**