

# Mathematics: analysis and approaches

## Standard level

### Paper 1 Review (Calculator Inactive)

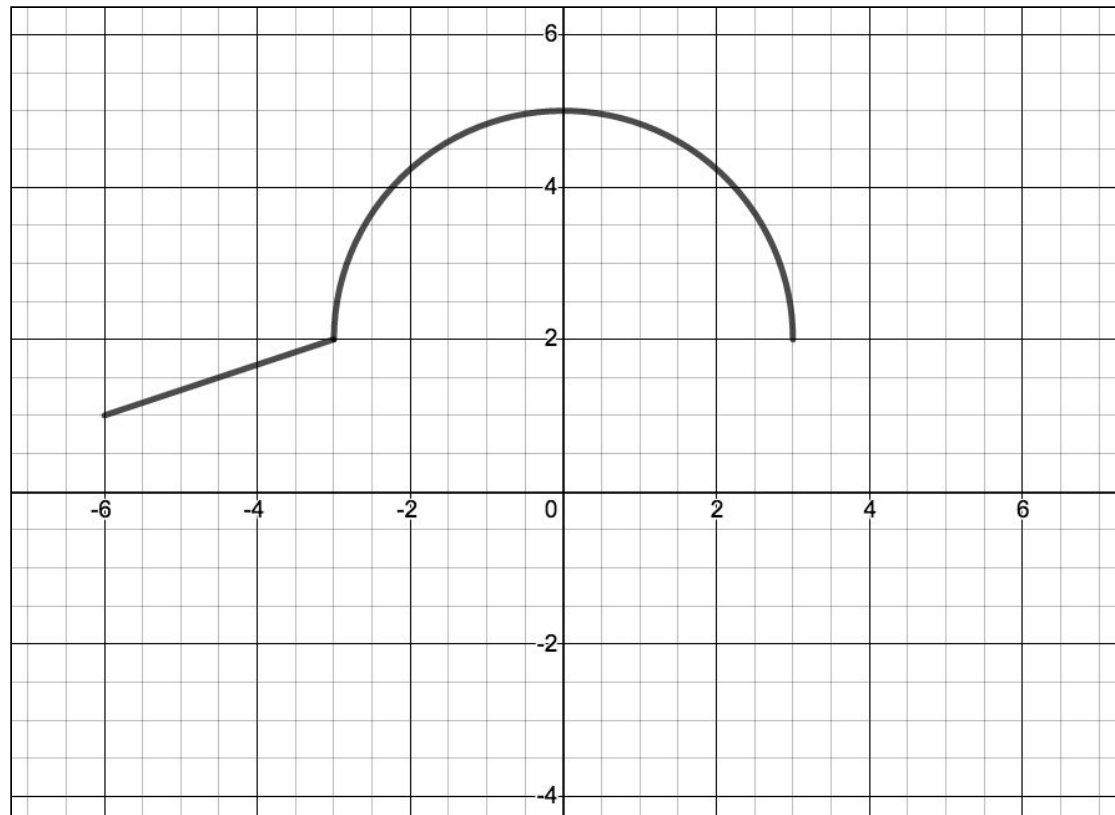
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

#### Section A

Answer all questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6] Find the equation of the line which passes through the origin (0,0) and the intersection of  $x + y = 2$ , and  $2x + 3y = 8$ .
2. [Maximum mark: 6] Mr. Yeo has a jar of marbles on his desk. There are 2 white, 3 blue, and 5 red marbles in the jar. Without looking, Jana takes one marble out of the jar, and then without looking Youssef takes a marble.
  - a. Draw a tree diagram to model this situation, be sure to label it properly.
  - b. What is the probability that Jana and Youssef got the same color marbles?
  - c. Given that Youssef drew a red marble, what is the probability that Jana got a white one?
3. [Maximum mark: 6] Solve the following equations for  $x$ ,
  - a.  $\log_2(3x + 4) = 4$ .
  - b.  $2^{x-3} = \frac{1}{16}$ .
  - c.  $\log_2(x^2 - 2x) = 3$ .
4. [Maximum mark: 5] The functions  $f$  and  $g$  are defined such that,
$$f(x) = \frac{x+3}{4} \text{ and } g(x) = 8x + 5.$$
  - a. Show that  $(g \circ f)(x) = 2x + 11$  [2 marks]
  - b. Given that  $(g \circ f)^{-1}(a) = 4$  find the value of  $a$ . [3 marks]
5. [Maximum mark: 6] Let  $\sin(\theta) = -\frac{4}{5}$ , and  $\cos(\theta) < 0$ .
  - a. In what quadrant is  $\theta$  located? [1 mark]
  - b. Determine the value of  $\tan(\theta)$ . [2 marks]
  - c. Let  $a = 5\cos(\theta) + 3\tan(\theta)$ . Write down the value of  $a$ . [3 marks]

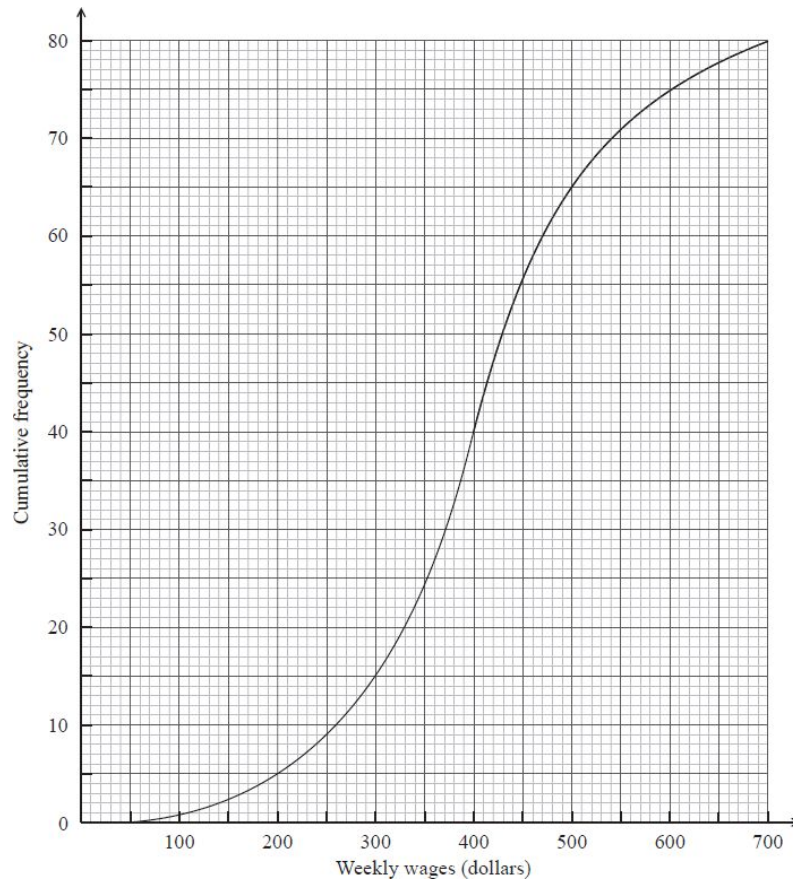
6. [Maximum mark: 5] The function  $f(x)$  is shown in the graph below.



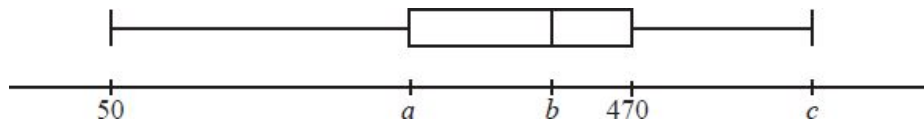
- a. Let  $g(x) = f(-2x) - 3$ . Draw a sketch of  $g(x)$  on the graph above. [3 marks]
  - b. The point  $A(-6, 1)$  is on  $f(x)$ . The point  $P$  is point  $A$  transformed onto  $g(x)$ . Write down the x- and y- coordinates of  $P$ . [2 marks]
7. [Maximum mark: 8] Given the following equations, solve for  $x$ ,
- a.  $\sqrt{3}\tan(x) + 1 = 0$ , where  $0 \leq x \leq 2\pi$  [4 marks]
  - b.  $2\cos(x) = \sqrt{3}$ , where  $0^\circ \leq x \leq 720^\circ$  [4 marks]
8. [Maximum mark: 10] Let  $A$  and  $B$  be events such that  $P(A) = 0.5$ ,  $P(B) = 0.4$ , and  $P(A \cup B) = 0.6$ .
- a. Draw a venn diagram representing this situation. [2 marks]
  - b. Find  $P(B \cap A)$ . [3 marks]
  - c. Find  $P(A | B)$ . [3 marks]
  - d. State whether  $A$  and  $B$  are independent events. Justify your answer. [2 marks]
9. [Maximum mark: 6] Given the expression  $3\ln 2 - \ln 4$ ,
- a. Write this expression in the form  $\ln k$ , where  $k \in \mathbb{Z}$ . [2 marks]
  - b. Hence or otherwise, solve  $3\ln 2 - \ln 4 = -\ln x$ . [4 marks]

## Section B

1. [Maximum mark: 15] The weekly wages (in USD) of 80 employees are displayed in the cumulative frequency curve below.



- a. Write down the median weekly wage [1 mark]
- b. Find the interquartile range of the weekly wages. [3 marks]



The box-and-whisker plot below displays the weekly wages of the employees.

- c. Write down the value of [3 marks]
  - i.  $a$ ;
  - ii.  $b$ ;
  - iii.  $c$ .
- d. Employees are paid 20 USD per hour.
  - i. Find the median number of hours worked per week. [3 marks]
  - ii. Find the number of employees who work more than 25 hours per week.

[5 marks]

2. [Maximum mark: 13] Let  $f(x) = \sqrt{x}$  and  $g(x) = 4 - x^2$ .

- a. Find;
  - i.  $f(g(2))$ . [1 mark]
  - ii.  $g(f(2))$  [1 mark]
- b. Write down the equation of  $f^{-1}(x)$ . [2 marks]
- c. Hence or otherwise, state the value(s) of  $x$  for which  $f^{-1}(x) = g(x)$ . [2 marks]
- d. If  $g \circ f^{-1}(a) = -12$ , write down the value of  $a$ . [2 marks]

The function  $h(x)$  is a line with a slope of  $-2$ , that passes through the point  $(-3, 7)$

- e. Determine an equation for the line  $h(x)$ . [2 marks]
- f. Determine the x-coordinate(s) of the point(s) of intersection between  $g(x)$  and  $h(x)$ . [3 marks]

3. [Maximum mark: 18] Given the quadratic equation  $f(x) = -2x^2 + 4x + 6$ .

- a. Determine the roots of the equation. [3]
- b. Find the axis of symmetry, and hence find the vertex of the equation [3]
- c. Write  $f(x)$  in vertex form, and from that determine the equation of the inverse of the function [3]
- d. Determine the equation of a line with slope of  $-2$  that intersects  $f(x)$  at the point  $(3, 0)$ . [2]
- e. The line from part d intersects  $f(x)$  at two points, determine the other point of intersection. [3]
- f. Let  $g(x) = x - 5$ . Let  $h(x) = f \circ g(x)$ . Determine  $h(3)$ . [2]
- g. Write the domain and range of  $h(x)$ . [2]

## Mathematics: analysis and approaches

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### Paper 2 Review (Calculator Active)

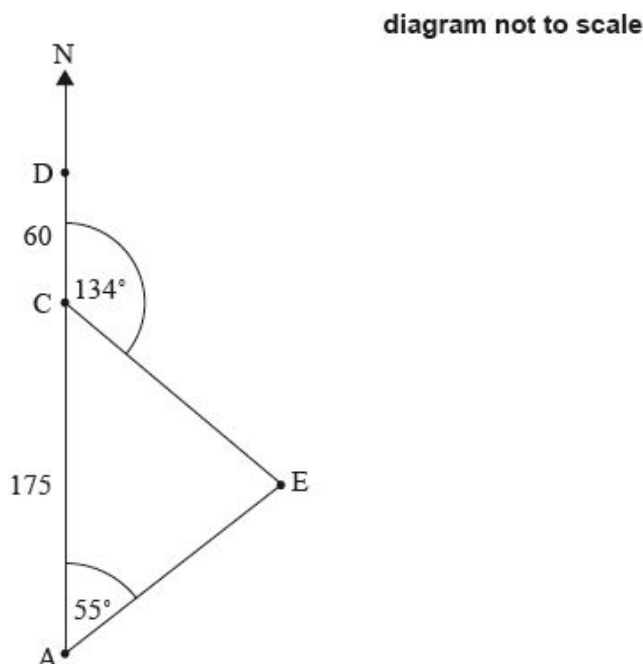
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

#### Section A

Answer all questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6] Given the binomial expression  $(2x - y)^6$ 
  - a. Determine the second term of the expansion of  $(2x - y)^6$
  - b. Write down the coefficient of the  $x^3y^3$  term of the expansion.
2. [Maximum mark: 6] In an arithmetic sequence, the fourth term is 16 and the eighth term is 40.
  - a. Find the first term and the common difference.
  - b. What is the 23rd term?
  - c. Find the sum of the first 23 terms.
3. [Maximum mark: 6] Given the functions  $f(x) = \sqrt{x-4}$  and  $g(x) = 3x + 2$ ,
  - a. State the domain and range of  $f \circ g(x)$ .
  - b. Determine the value of  $f \circ g(6)$ .
  - c. Determine the value of  $g^{-1} \circ f(40)$ .
4. [Maximum mark: 6] Given the following geometric sequence: 4, 12, 36, ...
  - a. Write down the value of  $r$  the common ratio, and  $U_1$  the first term. [2 marks]
  - b. Write down the value of the 9th term. [2 marks]
  - c. Determine the sum of the first 9 terms of this sequence. [2 marks]

5. [Maximum mark: 10] A ship is sailing north from a point A towards point D. Point C is 175km north of A. Point D is 60 km north of C. There is an island at E. The bearing of E from A is  $055^\circ$ . The bearing of E from C is  $134^\circ$ . This is shown in the diagram below.



- Find the bearing from E to A. [2 marks]
  - Find the distance CE. [5 marks]
  - Find the distance DE. [3 marks]
6. [Maximum mark: 8]
- Consider the following sequences:
- $$u_n = \{10, 20, 40, 80, 160, \dots\}$$
- $$v_n = \{10, 20, 30, 60, 100, \dots\}$$
- $$w_n = \{10, 20, 30, 40, 50, \dots\}$$
- State which sequence is arithmetic. [1 mark]
  - State which sequence is geometric. [1 mark]
  - Find the exact value of the 11th term of the geometric sequence [2 marks]
  - Find the sum of the first 20 terms of the arithmetic sequence. [2 marks]
  - For what value of n is the sum of the first n terms of the geometric sequence greater than 10,000? [2 marks]

7. [Maximum mark: 6] Money Boxes are coin containers used by children and come in a variety of shapes. The money box shown is in the shape of a cylinder. It has a radius of 4.43cm and a height of 12.2 cm.

diagram not to scale



- a. Find the volume of the money box. [3 marks]

diagram not to scale



- b. A second money box is in the shape of a sphere and has the same volume as the cylindrical box. Find the diameter of the second money box. [3 marks]
8. [Maximum mark: 6] The temperature  $A^{\circ}\text{C}$  inside a house at time  $t$  hours after 4:00am is given by

$$A = 21 - 3\cos\left(\frac{\pi}{12}t\right), \quad 0 \leq t \leq 24$$

and the temperature  $B^{\circ}\text{C}$  outside the house at the same time is given by

$$B = 22 - 5\cos\left(\frac{\pi}{12}t\right), \quad 0 \leq t \leq 24$$

- Find the temperature inside the house at 8:00am. [2 marks]
- Write down an expression for  $D = A - B$ , the difference between the inside and outside temperatures. [2 marks]
- Hence or otherwise, determine when the inside temperature first drops below the outside temperature. [2 marks]

## Section B

Answer all questions on the answer sheets provided. Round answer to three significant figures.

1. [Maximum mark: 18] There are three towns nearby each other. Boatsville is 15 km due north of Acorn City, and Candytown is 32 km north-east of Acorn City (at a bearing of  $045^\circ$ ).
- Draw a triangle representing the locations of Acorn City, Boatsville, and Candytown labelling the towns A, B, and C respectively. Making sure to label all distances and angles. [3]
  - Calculate the distance between Boatsville and Candytown. [3]
  - Determine the angle BCA. [3]
  - Determine the area of the triangle formed by connecting all three cities. [3]

An airplane is flying directly over Boatsville at a height of 3 km.

- At the moment the plane is right over Boatsville, determine how far the plane is from Candytown. [3]
  - At the same moment, Abdullah is in Candytown when he looks up and sees the airplane off in the distance. At what angle of elevation is Abdullah looking at the plane? [3]
2. [Maximum mark: 17] A healthy body temperature is  $37.0^\circ\text{C}$ . Eight people were medically examined and the difference in their body temperature ( $^\circ\text{C}$ ), from  $37.0^\circ\text{C}$ , was recorded. Their heartbeats (beats per minute) were also recorded.

|                                                    |      |     |      |      |      |    |     |     |
|----------------------------------------------------|------|-----|------|------|------|----|-----|-----|
| Temp. difference from $37.0^\circ\text{C}$ ( $x$ ) | -0.2 | 0.3 | -0.3 | -0.2 | -0.1 | 0  | 0.2 | 0.5 |
| Heartbeat ( $y$ )                                  | 63   | 77  | 70   | 74   | 65   | 78 | 79  | 86  |

- Draw a scatter diagram for the temperature difference from  $37.0^\circ\text{C}$  ( $x$ ) against heartbeat ( $y$ ). [4 marks]
- Use your calculator to find the correlation coefficient,  $r$ . [2 marks]
- Hence describe the correlation between temperature difference from  $37.0^\circ\text{C}$  and heartbeat. [2 marks]
- Use your calculator to find the equation of the regression line for this data set. [2 marks]
- Draw the regression line on the scatter diagram. [2 marks]
- Using your regression line, what would you expect the heartbeat to be for someone with a body temperature of  $37.4^\circ\text{C}$ . [2 marks]
- Using your regression line, what would you expect the heartbeat to be for someone with a body temperature of  $36.0^\circ\text{C}$ . Is this answer reasonable? Why/Why not? [3 marks]

3. [Maximum mark: 12] When we work with annuities, we use a specific form of the sum of a *geometric sequence formula*.

- Recall that the general form for the sums of any geometric series is

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

- When we are building up the money in an annuity we may use the FV equation.

$$FV_{annuity} = C * \left[ \frac{(1+i)^n - 1}{i} \right] \text{ Where: } C = \text{the payment per period, } i = \text{interest}$$

rate per period, and  $n$  = number of periods.

- When we are withdrawing money in retirement, we may use the PV equation

$$PV_{annuity\ due} = C * \left[ \frac{1 - (1+i)^{-n}}{i} \right] * (1 + i) \text{ Where } C = \text{the withdrawal per period, } i =$$

interest rate per period, and  $n$  = number of periods.

- At the age of 25 Salma received a lump-sum gift of 100,000 USD. She immediately invested in into an account earning interest of 4.2% p.a. If this account is compounded monthly, what is the value of this account when Salma retires at the age of 65? [3 marks]
- Salma's friend Omar decides to start saving money each month in an account earning 4.8% interest p.a. If he starts at 25 and also goes until he is 65, what amount must he contribute every month in order to end up with the same amount of money as Salma when they both retire? [3 marks]
- Salma wants her money to last for a long time after she retires. What is the maximum amount she can withdraw monthly to last her 25 years? [3 marks]
- Omar wants to live a fancy life. He is going to withdraw \$6,000 a month from his retirement account. How many years (to two decimal places) will his money last? [3 marks]