# Topic 4—Vectors 16 hours

The aim of this topic is to provide an elementary introduction to vectors, including both algebraic and geometric approaches. The use of dynamic geometry software is extremely helpful to visualize situations in three dimensions.

| | Content | Further guidance | Links |
|---|---|---|---|
| **4.1** | Vectors as displacements in the plane and in three dimensions.<br><br>Components of a vector; column representation; $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$ .<br><br>Algebraic and geometric approaches to the following:<br><br>• the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;<br><br>• multiplication by a scalar, $k\boldsymbol{v}$; parallel vectors;<br><br>• magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;<br><br>• unit vectors; base vectors; $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$;<br><br>• position vectors $\vec{\mathrm{OA}} = \boldsymbol{a}$ ;<br><br>• $\vec{\mathrm{AB}} = \vec{\mathrm{OB}} - \vec{\mathrm{OA}} = \boldsymbol{b} - \boldsymbol{a}$ . | Link to three-dimensional geometry, $x$, $y$ and $z$-axes.<br><br>Components are with respect to the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ (standard basis).<br><br>Applications to simple geometric figures are essential.<br><br>The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w})$ . Vector sums and differences can be represented by the diagonals of a parallelogram.<br><br>Multiplication by a scalar can be illustrated by enlargement.<br><br>Distance between points A and B is the magnitude of $\vec{\mathrm{AB}}$ . | **Appl:** Physics 1.3.2 (vector sums and differences) Physics 2.2.2, 2.2.3 (vector resultants).<br><br>**TOK:** How do we relate a theory to the author? Who developed vector analysis: JW Gibbs or O Heaviside? |

| | Content | Further guidance | Links |
|---|---|---|---|
| **4.2** | The scalar product of two vectors.<br><br>Perpendicular vectors; parallel vectors.<br><br>The angle between two vectors. | The scalar product is also known as the "dot product".<br>Link to 3.6, cosine rule.<br>For non-zero vectors, $\boldsymbol{v} \cdot \boldsymbol{w} = 0$ is equivalent to the vectors being perpendicular.<br>For parallel vectors, $\boldsymbol{w} = k\boldsymbol{v}$, $\lvert \boldsymbol{v} \cdot \boldsymbol{w} \rvert = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert$. | |
| **4.3** | Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.<br><br>The angle between two lines. | Relevance of $\boldsymbol{a}$ (position) and $\boldsymbol{b}$ (direction).<br>Interpretation of $t$ as time and $\boldsymbol{b}$ as velocity, with $\lvert \boldsymbol{b} \rvert$ representing speed. | **Aim 8:** Vector theory is used for tracking displacement of objects, including for peaceful and harmful purposes.<br>**TOK:** Are algebra and geometry two separate domains of knowledge? (Vector algebra is a good opportunity to discuss how geometrical properties are described and generalized by algebraic methods.) |
| **4.4** | Distinguishing between coincident and parallel lines.<br>Finding the point of intersection of two lines.<br>Determining whether two lines intersect. | | |