Topic 3—Circular functions and trigonometry 16 hours

The aims of this topic are to explore the circular functions and to solve problems using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated.

	Content	Further guidance	Links
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as exact multiples of π, or decimals.	**Int:** Seki Takakazu calculating π to ten decimal places. **Int:** Hipparchus, Menelaus and Ptolemy. **Int:** Why are there 360 degrees in a complete turn? Links to Babylonian mathematics. **TOK:** Which is a better measure of angle: radian or degree? What are the "best" criteria by which to decide? **TOK:** Euclid's axioms as the building blocks of Euclidean geometry. Link to non-Euclidean geometry.
3.2	Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$. Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.	The equation of a straight line through the origin is $y = x\tan\theta$. *Examples*: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan 210° = \frac{\sqrt{3}}{3}$.	**Aim 8:** Who really invented "Pythagoras' theorem"? **Int:** The first work to refer explicitly to the sine as a function of an angle is the Aryabhatiya of Aryabhata (ca. 510). **TOK:** Trigonometry was developed by successive civilizations and cultures. How is mathematical knowledge considered from a sociocultural perspective?

	Content	Further guidance	Links
3.3	The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double angle identities for sine and cosine.	Simple geometrical diagrams and/or technology may be used to illustrate the double angle formulae (and other trigonometric identities).	
	Relationship between trigonometric ratios.	*Examples*: Given $\sin\theta$, finding possible values of $\tan\theta$ without finding θ. Given $\cos x = \frac{3}{4}$, and x is acute, find $\sin 2x$ without finding x.	
3.4	The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs.		**Appl:** Physics 4.2 (simple harmonic motion).
	Composite functions of the form $f(x) = a\sin\left(b(x+c)\right) + d$.	*Examples*: $f(x) = \tan\left(x - \frac{\pi}{4}\right)$, $f(x) = 2\cos\left(3(x-4)\right) + 1$.	
	Transformations.	*Example*: $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the y-direction and a stretch of scale factor $\frac{1}{2}$ in the x-direction. Link to 2.3, transformation of graphs.	
	Applications.	Examples include height of tide, motion of a Ferris wheel.	

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3.5	Solving trigonometric equations in a finite interval, both graphically and analytically.	*Examples*: $2\sin x = 1$, $0 \le x \le 2\pi$, $2\sin 2x = 3\cos x$, $0^\circ \le x \le 180^\circ$, $2\tan(3(x-4)) = 1$, $-\pi \le x \le 3\pi$.	
	Equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$. **Not required:** the general solution of trigonometric equations.	*Examples*: $2\sin^2 x + 5\cos x + 1 = 0$ for $0 \le x < 4\pi$, $2\sin x = \cos 2x$, $-\pi \le x \le \pi$.	
3.6	Solution of triangles.	Pythagoras' theorem is a special case of the cosine rule.	**Aim 8:** Attributing the origin of a mathematical discovery to the wrong mathematician. **Int:** Cosine rule: Al-Kashi and Pythagoras.
	The cosine rule. The sine rule, including the ambiguous case.	Link with 4.2, scalar product, noting that: $\boldsymbol{c} = \boldsymbol{a} - \boldsymbol{b} \Rightarrow \lvert\boldsymbol{c}\rvert^2 = \lvert\boldsymbol{a}\rvert^2 + \lvert\boldsymbol{b}\rvert^2 - 2\boldsymbol{a}\cdot\boldsymbol{b}$.	
	Area of a triangle, $\frac{1}{2}ab\sin C$.		
	Applications.	Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.	**TOK:** Non-Euclidean geometry: angle sum on a globe greater than 180°.