

Syllabus content

Topic 1—Algebra

9 hours

The aim of this topic is to introduce students to some basic algebraic concepts and applications.

	Content	Further guidance	Links
1.1	<p>Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series.</p> <p>Sigma notation.</p> <p>Applications.</p>	<p>Technology may be used to generate and display sequences in several ways.</p> <p>Link to 2.6, exponential functions.</p> <p>Examples include compound interest and population growth.</p>	<p>Int: The chess legend (Sissa ibn Dahir).</p> <p>Int: Aryabhata is sometimes considered the “father of algebra”. Compare with al-Khawarizmi.</p> <p>TOK: How did Gauss add up integers from 1 to 100? Discuss the idea of mathematical intuition as the basis for formal proof.</p> <p>TOK: Debate over the validity of the notion of “infinity”: finitists such as L. Kronecker consider that “a mathematical object does not exist unless it can be constructed from natural numbers in a finite number of steps”.</p> <p>TOK: What is Zeno’s dichotomy paradox? How far can mathematical facts be from intuition?</p>

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1.2	<p>Elementary treatment of exponents and logarithms.</p> <p>Laws of exponents; laws of logarithms.</p> <p>Change of base.</p>	<p><i>Examples:</i> $16^{\frac{3}{4}} = 8$; $\frac{3}{4} = \log_{16} 8$;</p> <p>$\log 32 = 5 \log 2$; $(2^3)^{-4} = 2^{-12}$.</p> <p><i>Examples:</i> $\log_4 7 = \frac{\ln 7}{\ln 4}$,</p> <p>$\log_{25} 125 = \frac{\log_5 125}{\log_5 25} \left(= \frac{3}{2} \right)$.</p> <p>Link to 2.6, logarithmic functions.</p>	<p>Appl: Chemistry 18.1 (Calculation of pH).</p> <p>TOK: Are logarithms an invention or discovery? (This topic is an opportunity for teachers to generate reflection on “the nature of mathematics”.)</p>
1.3	<p>The binomial theorem: expansion of $(a + b)^n$, $n \in \mathbb{N}$.</p> <p>Calculation of binomial coefficients using Pascal’s triangle and $\binom{n}{r}$.</p> <p>Not required: formal treatment of permutations and formula for ${}^n P_r$.</p>	<p>Counting principles may be used in the development of the theorem.</p> <p>$\binom{n}{r}$ should be found using both the formula and technology.</p> <p><i>Example:</i> finding $\binom{6}{r}$ from inputting</p> <p>$y = 6^n C_r X$ and then reading coefficients from the table.</p> <p>Link to 5.8, binomial distribution.</p>	<p>Aim 8: Pascal’s triangle. Attributing the origin of a mathematical discovery to the wrong mathematician.</p> <p>Int: The so-called “Pascal’s triangle” was known in China much earlier than Pascal.</p>