Math SL PROBLEM SET 57

Section A (Short Answer) (NOTE: All Qs are CA)

1. In an arithmetic sequence, the third term is 41 and the ninth term is 23.
    a. Find the common difference.
    b. Find the first term.
    c. Find the smallest value of n such that $S_n < 0$.

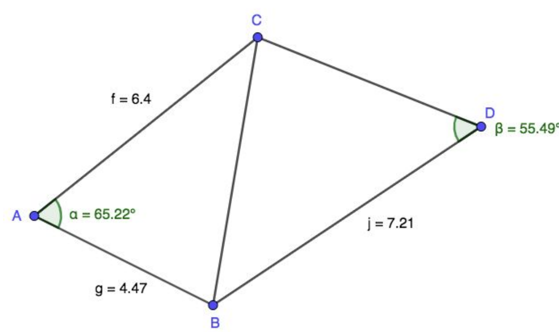
2. The maximum temperature T, in °C , on six randomly selected days is shown in the following table. The table also shows the number of soda cans purchased, N, from a vending machine.

Maximum temperature (T)	5	6	18	32	29	12
Number of soda cans (N)	26	28	37	41	48	29

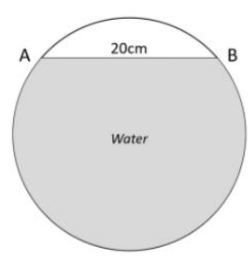
The relationship between the variables can be modelled by the regression equation $N = aT + b$.
    a. Find the value of r, the correlation coefficient.
    b. Find the value of a and b.
    c. Hence, use the regression equation to estimate the number of soda cans purchased on a day where the maximum temperature is 23°C.

3. Let $f(x) = x^3 + 1$ and $g(x) = x - 2$ for $x \in \mathbb{R}$.
    a. Find $f(2)$. b. Find $f^{-1}(x)$. c. Solve $(f \circ g)(x) = 0$.

4. Given quadrilateral ABCD, determine:
    a. the measure of the diagonal BC
    b. the measure of $\angle$BCD



5. A bag contains 8 blue marbles, 12 green marbles and m red marbles. A marble is selected at random and replaced. This is performed 3 times.
    a. Write down the probability that the first marble selected is blue.
    b. Let X be the number of blue marbles selected. Find the smallest value of m for which $\text{Var}(X) < 0.5$



6. The diagram below shows a cylindrical pipe, 80 cm in length, carrying water. The pipe has a radius of 15 cm. The pipe is not at full capacity, such that the chord length of the water level [AB] is 20 cm. Find the volume of water in the pipe.

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7. Let $f(x) = (x^2 + k)^5$.
   a. Write down the expansion of $f(x)$.
   b. Hence or otherwise, in the expansion of the derivative, $f`(x)$, the coefficient of the term in x^5 is 960. Find the possible values of k.

Section B (Extended Response) (ALL Qs are CA)

8. A particle P moves in a straight line for 6 seconds. Its acceleration during this period is given by $a(t) = -2t^2 + 13t - 15$, for $0 \leq t \leq 6$.

   a. Write down the value(s) for t when the particle's acceleration is zero.
   b. Hence, or otherwise, find all possible values for t for which the velocity of P is increasing.

   The particle has an initial velocity of 7 m/s.

   c. Find an expression for the velocity of P at time t.
   d. Find the total distance travelled by P when its velocity is decreasing.

9. Tom enters a tennis tournament where he plays three matches every day. The results of each match are independent of each other.

   Let A be the number of matches Tom wins on any given day of the competition. The probability distribution for A can be modelled by the following table:

a	0	1	2	3
$P(A = a)$	0.15	0.2	p	0.25

   a. Find the value of p.
   b. (i) A day is chosen at random. Write down the probability that Tom wins every match.
   (ii) The competition goes on for 4 days. Find the probability that Tom wins every match on exactly three of these days.

   Clare enters the same tennis competition. B is the number of matches Clare wins on any given day of the competition. The probability distribution for B can be modelled by the following table:

b	0	1	2	3
$P(B = b)$	0.05	0.1	0.35	0.5

   c. Find E(B)

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On the final day of the competition, both Tom and Clare play their respective matches and their results are independent. The number of wins Tom and Clare obtain are added together to form a total out of six.

d. (i) Find the probability that they win more than 4 matches combined.
   (ii) Given that they won more than 4 matches, find the probability that Clare won all three of her matches.

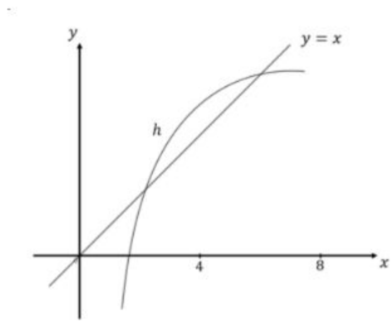
10. Let $f(x) = \ln(x)$ and let $g(x) = 2 + 3\ln(x - 1)$ for $x > 1$. The graph of $g(x)$ can be obtained from the graph of f by two transformations: a vertical stretch of scale factor q and a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$

   a. Write down the values of q, h and k.

   Let $h(x) = g(x) \times \cos(0.1x)$ for $1 < x < 80$.

   b. Determine the point(s) on $h(x)$ where:
      i. $h'(\mathrm{x}) = 0$ and $h''(x) < 0$
      ii. $h'(x) = 0$ and $h''(x) = 0$

   The diagram shows the graph of $h(x)$ and the line $y = x$ on the domain of $1 < x < 8$.



   c. Solve the equation $h(x) = x$.

   d. Let d be the vertical distance from a point on the graph of h to the line $y = x$. There is a point $Q(x,y)$ on h where d is a maximum. Find the coordinates of Q where Q is between the two intersection points between h and the line $y = x$.