Math SL PROBLEM SET 56

Section A (Short Answer) (NOTE: All Qs are CI)

1. Events A and B are independent with $P(A \cap B) = P(A \cap B') = 0.3$
   a. Find $P(A)$.
   b. Find $P(A \cup B)$.
   c. Draw a Venn diagram for this situation.
   d. Draw a tree diagram for this situation

2. Let $\sin(\theta) = \frac{2}{3}$, where θ is obtuse. Find $\cos(\theta)$ and hence or otherwise, find $\cos(2\theta)$.

3. Let $f(x) = e^x - 2$ and $g(x) = 3 + \frac{2}{x}$, for $x \in \boldsymbol{R}$.
   a. Find $(g \circ f)(x)$.
   b. Find the exact value of the vertical asymptote of $(g \circ f)(x)$.
   c. Find $\lim_{x \to -\infty} (g \circ f)(x)$.

4. Consider the function $f(x) = \ln(2x - 1)$. Let point A be a point on the curve where $x = 3$.
   a. Determine the gradient (slope) of the curve at A.
   b. The normal to the curve at A cuts the x-axis at P. Find the coordinates of P.

5. The first three terms of a geometric sequence are $\ln(x^9)$, $\ln(x^3)$, $\ln(x)$ for $x > 0$.
   a. Find the common ratio.
   b. Solve the equation $27 = \sum_{k=1}^{\infty} 3^{3-k} \ln x$.

6. Solve $log_{\sqrt{3}}(\sin x) - log_{\sqrt{3}}(\cos x) = 1$ for $0 < x < \frac{\pi}{2}$.

7. The line L passes through the points $P(5,0)$ and $Q(0,2)$. The origin is at O. The point $S(3 - 4x, x)$ is on L, and the line OS is perpendicular to L.
   a. Write down the vectors $\langle PQ \rangle$ and $\langle OS \rangle$.
   b. Find the coordinates of the point S.

Math SL PROBLEM SET 56

Section B (Extended Response/Investigation)

8. Let $f(x) = 2x^3 + 3x^2 - 12x + 10$ for $x \in \boldsymbol{R}$.
   a. Find the x-coordinates of the extrema.
   b. Use the second derivative to classify the extrema as maximums, minimums or neither.
   c. Evaluate $F(b) - F(a)$ where $F(x)$ is the antiderivative of $f(x)$ and a and b are the x-coordinates of the extrema from Q(a) (NOTE: $b > a$).

9. A quadratic function f can be written in the form $f(x) = a(x - 1)(x - b)$. The graph of f has an axis of symmetry at $x = 2$ and a y-intercept at (0,-3).

   a. Find the value of b.
   b. Find the value of a.
   c. The line $y = kx + 6$ is a tangent to the curve of f. Find the values of k.
   d. The quadratic function, $f(x)$, is the derivative of some other function, $F(x)$. If $F(3) = 2$, find the equation of $F(x)$.

10. A line L_1 passes through the points A(0,-3,1) and B(-2,5,3). Show that $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$
    a. A second line, L_2, has equation $r = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. The lines L_1 and L_2 intersect at point C. Find the coordinates of C.

11. The following table shows the probability distribution of a discrete random B, in terms of an angle θ.

b	0	1
$P(B = b)$	$2\cos(2\theta)$	$\cos(\theta)$

   a. Show that $\cos\theta = \frac{3}{4}$.
   b. Find $\tan\theta$, given that $\tan\theta > 0$.