Math SL PROBLEM SET 52

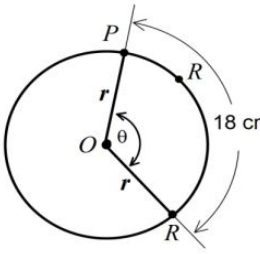
Section A (Skills/Concepts Consolidation)

1. **(F2.1, F2.2, F2.4 - R) (CI)** Given the function $g(x) = x^2 + 2x + 3$, where $x > -1$.

(Cirrito 5.4, p148)

   a. Use calculus to find the vertex of $g(x)$.
   b. Hence, or otherwise, find the inverse, $g^{-1}(x)$.
   c. On the same set of axes, sketch the graphs of $g(x)$ and $g^{-1}(x)$, labeling all intercepts.
   d. Will there exist a value of x such that $g(x) = g^{-1}(x)$? If so, find its value. If not, explain why not.

2. **(T3.1 - R) (CA)** The diagram shows a circle with a radius of r and its center at O. The central angle $\angle POR$ measures θ radians. The length of the minor arc PR is 18 cm. The area of the sector $OPSR$ is 108 cm^2. Find the value of r and the value of θ.



(Cirrito 9.7, p309)

3. **(A1.3 - N) (CA)** Consider the expression $\left(\frac{3}{x} - 2x^2\right)^6$,

(Cirrito 4.1.2, p100)

   a. Find the first three terms of this expansion.
   b. Find the coefficient of the x^9 term OR justify that it does not exist.
   c. Find the constant term of this expansion OR justify that it does not exist.

4. **(F2.5 - R) (CI)** For the rational function $r(x) = \frac{2x-5}{x-2}$, $x \neq 2$, determine: **(Cirrito 5.3.5, p144)**

   a. the equation(s) of the asymptote(s) and the intercepts of $r(x)$.
   b. Evaluate $\lim_{x \to \infty} r(x)$.
   c. Rewrite the equation of $r(x)$ in the form $r(x) = a + \frac{b}{x-2}$; $a, b \in Z$.
   d. Hence, determine the transformations that were applied to $y = \frac{1}{x}$ to create $r(x)$.

5. **(C6.3 - N) (CI)** Maylis knows that the derivative of a function is $\frac{dy}{dx} = (x+2)(x-3)(e^x - 1)$. So, she needs help in determining the: **(Cirrito 20.2, p649)**

   a. x-coordinates of the extrema of the original function, $y = f(x)$.
   b. intervals of increase and decrease of the original function.
   c. classification of the extrema as being either maximums or minimums or neither.

Math SL PROBLEM SET 52

Section B (Skills/Concepts Practice)

6. **(C6.2, C6.6 - N) (CI)** The position at t seconds of a particle moving along a **straight line (i.e. forwards or backwards)** is given by $s(t) = 3t^3 - 40.5t^2 + 162t$, where s is measured in meters and $t \geq 0$. **(Cirrito 21.3, p694)**

   a. Find the position at $t = 4$ s.
   b. Is the particle moving forwards or backwards at $t = 4$? How do you know?
   c. Determine the speed of the particle at $t = 2$ s.
   d. Determine the average speed in the first 3 seconds of travelling.
   e. Determine when the particle's speed is increasing
   f. Determine when the particle's acceleration is 0.

7. **(F2.2, F2.4, F2.5, C6.1) (CI)** Let $f(x) = 1 - \frac{3}{x}$ and let $g(x) = x^2 - 3x$.

   a. Find, if possible, the coordinates of the minimum point(s) of f and g OR explain why it is not possible.
   b. Find the coordinates of the point(s) of intersection of the graphs of f and g.
   c. Find the equation of the inverse function of each function.
   d. Find the equations of the line tangent to $g(x)$ and the line tangent to $f(x)$ at $x = 1$.
   e. Where do the two tangent lines intersect?

8. **(C6.1 - N) (CA)** Given the function $g(x) = \frac{1}{x+3}$, **(Cirrito 18.3, p592)**

   a. determine the value of $g(1)$ as well as determining an expression for $g(1 + h)$
   b. and hence, determine an expression for the difference quotient, $\frac{g(1+h) - g(1)}{h}$
   c. What does $\lim_{h \to 0} \frac{g(1+h) - g(1)}{h}$ represent, geometrically?
   d. What would you predict the derivative of $g(x) = \frac{1}{x+3}$ to be?

9. **(SP5.2, SP5.3, SP5.6 - R) (CA)** The length of 80 flower stems in Mr Smith's garden are shown in the following cumulative frequency diagram (at the end of the question, on the next page) **(Oxford 8.5, p271)**

   a. Write down the median length.
   b. What percentage of flower stems are 60 cm or greater?
   c. At least 18.75% of all flowers have a flower stem length of K. Find the value of K.

Math SL PROBLEM SET 52

The same data is now presented as a frequency table.

Length, x cm	$0 \leq L \leq 30$	$30 < L \leq 60$	$60 < L \leq 90$	$90 < L \leq 120$
Frequency	10	p	20	q

d. Find the value of p and q.
e. Hence, estimate the mean and standard deviation of the lengths.

Flower stems that are 60 cm or greater in length are considered mature flowers.

f. Given that a randomly selected flower is mature, find the probability that its stem length is 85 cm or greater in length.