Math SL PROBLEM SET 51

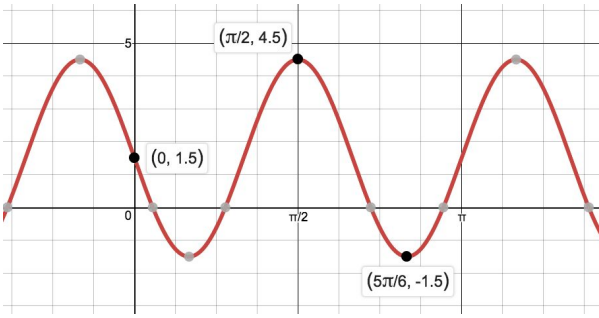
Section A (Skills/Concepts Consolidation)

1. **(A1.2 - N)** **(CI)** Use the properties of logarithms to write each logarithmic expression as a sum, difference or constant multiple of single logarithms **(Cirrito 7.4, p221)**

   a. $\log_2(2m)$ b. $\ln \sqrt[5]{x}$ c. $\log_3(a^2b^3)$ d. $\log_{10}[10x(1+r)^t]$ e. $\ln\left(\frac{m^3}{n}\right)$

2. **(T3.4 - R)** **(CI)** The function, $y = f(x)$ on the domain of $0 \le x \le \pi$, is pictured. Determine: **(Cirrito 10.3, p337)**

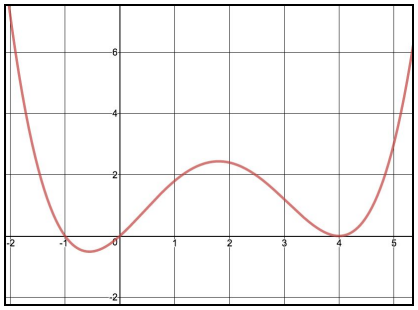
   a. the amplitude, the period and the axis of the curve.
   b. an appropriate equation for the function.
   c. the intervals of increase on the domain of $0 \le x \le \pi$.
   d. the exact values of the zeroes on $0 \le x \le \pi$.
   e. where is $\frac{d}{dx} f(x) = 0$?



3. **(F2.6, F2.2 - R)** **(CI)** A function is defined as $g(x) = e^x - 1$. **(Cirrito 5.3.3, p136; 5.3.4, p141)**

   a. Find the intercept(s) and asymptote(s) of g.
   b. Hence, sketch the function.
   c. Mr. S. suggests that $f(x) = ln(x) + 1$ is the inverse of g. Simplify the composition $(g \circ f)(x)$ to see if Mr. S is/isn't correct.

4. **(CA6.3 - N)** **(CI)** Here is the graph of the **derivative of a function**, $\frac{d}{dx} f(x)$. List what you can figure out about the original function and then prepare a sketch of the original function.



5. **(P5.6 - R)** **(CI)** Two events, A and B, are such that $P(A) = \frac{9}{16}$ and $P(B) = \frac{3}{8}$ and $P(A \mid B) = \frac{1}{4}$. Find the probability that: **(Oxford 3.4, p85)**

   a. Both events will happen.
   b. Only one of the events will happen
   c. Neither of the events will happen
   d. Event A happens given that both events happen.

Math SL PROBLEM SET 51

6. **(SP5.7 - N)** **(CA)** A discrete random variable X has the following probability distribution.

X	0	1	2	3
$P(X=x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

   a. Find the value of k.
   b. Write down $P(X=2)$.
   c. Find $P(X=2 \mid X>0)$.

Section B (Skills/Concepts Practice)

7. **(F2.1, F2.3, C6.1 - R,N)** **(CI)** Consider the function $g(x)=\sqrt{x+4}$, **(Cirrito 6.1, 6.2)**

   a. Determine the domain and range of $y=g(x)$.
   b. The function $y=g(x)$ is now translated 6 units to the left and then horizontally compressed by a factor of 3. Write down the new equation of this transformed function.
   c. Determine the equation of $y=g^{-1}(x)$.
   d. What is the domain of $y=g^{-1}(x)$?
   e. Determine the simplified equation that results from the $\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$ calculation.
   f. Hence or otherwise, determine the rate of change of $y=g(x)$ at the point where $x=12$.

8. **(V4.3, T3.6 - N,R)** **(CA)** The triangle ABC has vertices at A(-1,2,3) and B(-1,3,5) and C(0,-1,1). **(Cirrito 12.6.1, p432)**

   a. Find the measure of the angle at vertex A.
   b. Hence, or otherwise, find the area of the triangle.

   Let L_1 be the line parallel to vector AB which passes through D(2,-1,0) and let L_2 be the line parallel to vector AC which passes through E(-1,1,1)

   c. Find the equations of L_1 and L_2
   d. Hence, show that L_1 and L_2 do NOT intersect

9. **(T3.2, CA6.3 - N)** **(CI)** The derivative of a function is $\frac{d}{dx}f(x)=x-2x\cos(x)$, where $0\le x\le 2\pi$.

   a. Factor this derivative equation: $\frac{d}{dx}f(x)=x-2x\cos(x)$
   b. Hence, determine where the original function, $f(x)$, has its extrema.
   c. Evaluate $\frac{d}{dx}f\left(\frac{\pi}{6}\right)$ and $\frac{d}{dx}f(\pi)$ and $\frac{d}{dx}f(2\pi)$.
   d. Determine the intervals in which the **original** function is (i) increasing, (ii) decreasing.
   e. Hence, sketch the **original** function from the information you have from the derivative.