Math SL PROBLEM SET 48

Section A (Skills/Concepts Consolidation)

1. **(V4.2 - N) (CA)** The lines L_1 and L_2 have the following equations: **(Cirrito 12.6.1, p432)**

   $L_1: x = 1 + 4t, y = 5 - 4t, z = -1 + 5t$ $\quad\quad$ $L_2: x = 2 + 8s, y = 4 - 3s, z = 5 + s$

   a. Is the point (-7,13,-16) on L_1?
   b. What is meant by the term "skew lines"?
   c. Show that these lines are skew.

2. **(A1.1 - E) (CA)** An arithmetic series has a first term of -4 and a common difference of 1. A geometric series has a first term of 8 and a common ratio of 0.5. After how many terms does the sum of the arithmetic series exceed the sum of the geometric series? **(Cirrito 8.2.3, p261)**

3. **(A1.2 - N) (CI)** Mr. S. would like to solve the equation $\log_4(x + 1) + \log_{\frac{1}{16}}(x + 1) = 1$. **(Cirrito 7.4, p244)**

   a. Explain why he cannot start by using the addition rule of logarithms.
   b. Re-express $\log_{\frac{1}{16}}(x + 1)$ in terms of log base 4 (i.e. $\log_4(??)$)
   c. Hence or otherwise, solve the equation $\log_4(x + 1) + \log_{\frac{1}{16}}(x + 1) = 1$.

4. **(C6.1, C6.3 - N) (CI)** Here is an equation of a cubic function, $g(x) = x^3 - 2x^2 - 3x - 5$. **(Cirrito 20.2, p649)**

   a. On what interval are the function values increasing?
   b. On what interval is the function concave up?
   c. What is the slope of this cubic function at $x = 4$? Explain how you determined this value.
   d. At what point(s) would you expect the slope of the tangent line(s) to be zero? Explain your reasoning.
   e. On what domain does the function have negative slopes? Explain your reasoning.
   f. What is the equation of the anti-derivative of $g(x)$?

5. **(A1.3 - N) (CA)** Consider the expression $\left(\frac{2}{x^2} - x\right)^7$, ***(Cirrito 4.1.2, p100)***

   a. Find the first three terms of this expansion.
   b. Find the last three terms of this expansion.
   c. Find the constant term of this expansion OR justify that it does not exist.

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6. **(SP5.8 - N) (CA)** A bag consists of 6 white cubes and 10 black cubes. Cubes are withdrawn one at a time, with replacement. Find the probability that after 4 draws **(Cirrito 16.3.4, p544)**

   a. all the cubes are black;
   b. there are at least 2 white cubes;
   c. there are at least 2 white cubes given that there was at least one white cube.

Section B (Skills/Concepts Practice)

7. **(SP5.7 - N) (CA)** Given the following table for a discrete random variable, X, (say the number of times this week that Mohamed is late to Math class) **(Cirrito C16.1, p533)**

x	1	2	3
$P(X = x)$	0.2	$1 - k$	

   a. Determine $P(X = 3)$. Express your answer in terms of k.
   b. What range of values can k take?
   c. Find, in terms of k, the mean of the distribution.
   d. Now suppose that $k = 0.35$. Find the mean and variance of the distribution.
   e. Now suppose that $k = 0.35$. Determine $P(X = 3 \mid X \geq 2)$

8. **(V4.2 - N) (CA)** Answer the following questions, involving the scalar (or dot) product: **(Cirrito 12.6, p440)**

   a. Find the measure of the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ if $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$.
   b. Find $(\boldsymbol{a} + 3\boldsymbol{b}) \cdot (2\boldsymbol{a} - \boldsymbol{b})$ where $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$.
   c. Given that $\boldsymbol{a} = 3\boldsymbol{i} - 5\boldsymbol{k}$ and given that $\boldsymbol{b} = 2\boldsymbol{i} + 7\boldsymbol{j}$ and given that $\boldsymbol{c} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, find the vector $\boldsymbol{d}$ such that $\boldsymbol{a} \cdot \boldsymbol{d} = -9$ and $\boldsymbol{b} \cdot \boldsymbol{d} = 11$ and $\boldsymbol{c} \cdot \boldsymbol{d} = 6$.

9. **(SP5.7, SP5.8 - N) (CI)** A fair six-sided dice has a "1" on one face, has a 2 on two of its faces and has a 3 on three of its faces. The dice is thrown twice. The random variable, T, represents the total score resulting from the two dice being thrown. **(Cirrito 16.3, p545)**

   a. Find $P(T = 3)$ and explain what the answer means in the context of the problem.
   b. Prepare a probability distribution table for this "experiment".
   c. Find the probability that the total score is more than 4.