Math SL PROBLEM SET 45

Section A (Skills/Concepts Consolidation)

1. **(CA6.1 - N) (CI)** Determine:

   (Cirrito 20.1, p646)

   a. the equation of the tangent line(s) to the curve of $y = x^3 - 3x^2 + 2x$ at the point(s) where the function has x-intercepts
   b. the equation of the tangent line to the curve of $y = x^2 - 2x$ that is perpendicular to the line $x - 2y = 1$.

2. **(CA6.1 - N) (CI)** Consider the function $g(x) = x^2(x - 1)$.

   (Cirrito 20.1, p646)

   a. Find the 2 points on g where the slope of the curve is 8.
   b. Find the equations of the tangents at both of these points.
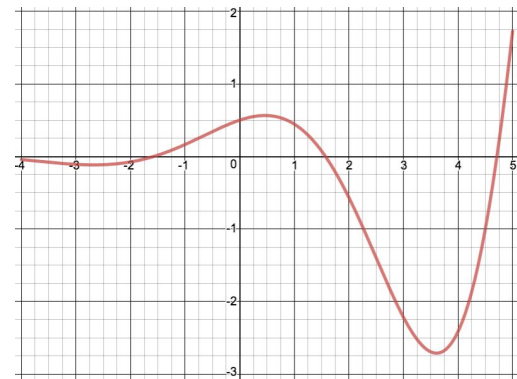
3. **(CA6.3 - N) (CI)** Here is a graph of a function. Draw graphs of the first and second derivatives of this function.

   (Cirrito 19.2, p609)

4. **(CA6.3 - N) (CI)** For $f(x) = x^4 - 2x^2$ determine:

   (Cirrito 20.2, p649)

   a. the equation of the derivative of $f(x)$.
   b. the zeroes of $f\grave{}$.
   c. Hence or otherwise, find the coordinates of the **stationary points** of f.
   d. Hence or otherwise, find the **intervals of increase and decrease** of f.
   e. Sketch a graph of f. Then use your calculator and graph f and compare.

5. **(CA6.3 - N) (CI)** For the function $f(x) = 2x^3 - 3x^2 - 12x$ determine:

   (Cirrito 20.2, p649)

   a. the equation of the second derivative of $f(x)$,
   b. the zeroes of $f\grave{}\grave{}$.
   c. Hence or otherwise, find the coordinates of the **inflection points** of f.
   d. Hence or otherwise, find the **intervals of concavity** of f.
   e. Sketch a graph of f. Then use your calculator and graph f and then compare.

6. **(V4.2 - N) (CA)** Find the value of k such that the lines $\frac{x-2}{k} = \frac{y}{2} = \frac{3-z}{3}$ and $\frac{x}{k-1} = \frac{y+2}{3} = \frac{z}{4}$ are perpendicular. Then, if possible, find the point at which the lines intersect.

   (Cirrito 12.6.1, p432)

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Section B (Skills/Concepts Practice)

7. **(V4.2 - N) (CA)** The position of two helicopters X and Y at time t seconds are given by the

    equations $r_X = \begin{pmatrix} 11 \\ 3 \\ -3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ and $r_Y = \begin{pmatrix} 1 \\ -7 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix}$. Distances are given in meters.

    (Cirrito 12.7.2, p452)

    a. Find the speed of the two helicopters.
    b. Show that the helicopters do NOT meet.
    c. Find the distance between the helicopters when $t = 10$ seconds.

8. **(SP5.7 - N) (CA)** For a discrete random variable, X, the probability distribution is defined by the

    equation : $P(X = x) = f(x) = \begin{cases} kx & x = 1, 2, 3, 4, 5 \\ k(10 - x) & x = 6, 7, 8, 9 \end{cases}$ Find:

    (Cirrito C16.1, p533)

    a. The value of the constant, k.
    b. Hence, find $P(X = 3)$
    c. Find the mean (now called the **expected value** of X)

9. **(F2.1, F2.4, C6.1 - R,E) (CI)** A quadratic function is given by the equation $f(x) = x^2 + 4x + B$.

    (Cirrito 5.4.2, p157)

    a. Determine the value of B if $f^{-1}(x) = -2 + \sqrt{x - 6}$.
    b. Perform the following compositions: (i) $f \circ f^{-1}(x)$ and (ii) $f^{-1} \circ f(x)$. Describe what happens and explain why.
    c. At what point would you expect the tangent line drawn to the quadratic function to have a zero slope? Explain why.