Math SL PROBLEM SET 39

Section A (Skills/Concepts Consolidation)

1. (**F2.6, F2.8 - R**) (**CA**) Which is best (and by how much?) - money in the bank that pays:
   i. 9% annual interest,
   ii. 9% interest compounded monthly,
   iii. 9% interest compounded daily?

   Show calculations to support your decision. **(Cirrito 7.1.5, p207)**

2. (**F2.6, F2.8 - R**) (**CA**) (CONTINUATION) Inflation in the country of Mathsylvania has reached alarming levels. Many banks are paying 100% interest compounded annually, some banks are paying 100% interest compounded monthly, a few are paying 100% interest compounded daily and so forth. In trying to make sense of all these different bank promotions, Daniel decides to graph the function $E(x) = \left(1 + \frac{1}{x}\right)^x$. What does the graph reveal about the sequence $v_n = E(n) = (1 + \frac{1}{n})^n$, where n is a positive integer? Calculate these specific values: v_1, v_{12}, v_{365}, $v_{3156000}$. **(Cirrito 7.1.5, p207)**

3. (**F2.6, F2.8 - R**) (**CA**) (CONTINUATION) The sequence in the previous problem has a limiting value. This sequence is so important that a special letter is reserved for the limiting value (as is done for π). We write $e = \lim_{n\to\infty}(1 + \frac{1}{n})^n$. The $\lim_{n\to\infty}$ means "as n approaches ∞" or "as n gets very large". For some additional work with this sequence, use your calculator to evaluate $\lim_{n\to\infty}(1 + \frac{0.09}{n})^n$. Make up a story to go with the question. **(Cirrito 7.1.5, p207)**

4. (**C6.1 - N**) (**CA**) Use calculator to draw tangent lines to following functions at the given points. For each function, include a sketch of the function with the tangent line, write down the equation of the tangent line, the slope of tangent as well as the meaning of the tangent line with respect to the function at that given point. **(Cirrito 18.3, p591)**

   a. The function $f(x) = 2x^2 + x - 1$ at $x = 3$
   b. The function $g(x) = \sin(x)$ at $x = \frac{\pi}{4}$
   c. The function $h(x) = 2e^x + 1$ at $x = \ln(3)$

5. (**C6.3 - N**) (**CA**) Graph the quartic polynomial $p(x) = -x^4 + 2x^2 - x + 1$ on your TI-84 and hence determine: **(Cirrito 20.2.2, p651)**
   a. the x-coordinate(s) of the extremas.
   b. the domain interval(s) in which the function values are **decreasing**.
   c. the x-coordinates of the inflection point(s).
   d. the domain interval(s) in which the function is **concave up**.
   e. Include a sketch, labelling the important points from (a) and (c).

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Section B (Skills/Concepts Practice)

6. **(V4.3 - N) (CA) SKILL**: Vector equations of Lines. The path of a boat sailing on the Red Sea is defined using the vector equation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, where the parameter, t, represents time measured in hours since leaving Hurghada and x,y are measured in kilometers.
   a. When $t = 7$, then $x = 26$ and $y = 26$. Explain what these numbers signify about the boat and its path travelled.
   b. Create a table of values (showing t,x and y) where $t \in \{0,1,2,3,4,5\}$.
   c. Explain the role of the parameter t.
   d. Determine the speed of the boat.
   e. Does this path take the boat to Treasure Island, positioned at $\begin{pmatrix} 35 \\ 40 \end{pmatrix}$?

7. **(V4.1, V4.3 - R) (CI) SKILL**: Vector equations of Lines. For each of the following pairs of points: (i) $A(3,4)$ and $B(7,-1)$; and (ii) $A(-2,3)$ and $B(5,1)$:
   a. Write $\overrightarrow{AB}$ in column form and in unit vector form and determine $\left|\overrightarrow{AB}\right|$.
   b. Write the vector $\frac{1}{\left|\overrightarrow{AB}\right|} \times \overrightarrow{AB}$ in component form. Explain the significance of this vector.
   c. Write a vector equation of a line going through the points A and B.

8. (**T3.5 - E**) **(CI) SKILL**: Quadratic Trig Equations & Identities. The equation $\cos^2(x) - \sin^2(x) = 0$ can be solved in a variety of ways: **(Cirrito 10.2.2, p332)**
   a. Solve $\cos^2(x) - \sin^2(x) = 0$ by factoring $\cos^2(x) - \sin^2(x)$
   b. Solve $\cos^2(x) - \sin^2(x) = 0$ using a Pythagorean identity
   c. Solve $\cos^2(x) - \sin^2(x) = 0$ using a tangent identity
   d. Solve $\cos^2(x) - \sin^2(x) = 0$ using a double angle identity

9. **(T3.5 - E) (CI) SKILL**: Quadratic Trig Equations & Identities. Each of these equations involves a double angle. Solve for x on the domain of $0 \leq x \leq 2\pi$: **(Cirrito 10.2.2, p332)**
   a. $\sin(2x) - \cos(x) = 0$ b. $\sin(x) - \cos(2x) = 0$

Section C (Skills/Concepts HW)

10. Vector Equations of Lines: Cirrito 12.7.1, p450, Q4ab , 5ab, 6ab, 7ab