Math SL PROBLEM SET 36

Section A (Skills/Concepts Consolidation)

1. (**A1.1, A1.2 - E**) (**CA**) A series is defined as $\log_2(3) + \log_2(3)^2 + \log_2(3)^3 + \log_2(3)^4 + \ldots\ldots$ What is the smallest value of n such that $S_n > 1000$?   **(Cirrito 7.4, p221)**

2. (**F2.7 - E**) (**CI**) Given the function $q(x) = 2x^2 - 3x + 2$, determine: **(Cirrito 2.4.1, p41)**

   a. The number of x-intercepts of $q(x)$.
   b. The value of K in the linear function $f(x) = -x + K$ such that the equation $q(x) = f(x)$ has only one solution.
   c. Interpret the meaning of the scenario in Q(b).

3. (**F2.6 - R**) (**CI**) Given the functions $f(x) = e^{2x+1}$ and $g(x) = \ln \sqrt{x}$;

   (Cirrito 5.3.3, p131; 5.3.4, p138)

   a. Sketch each function.
   b. Show that $(g \circ f)(x) = x + \frac{1}{2}$. Hence, are the 2 functions inverses of each other?
   c. Find the equation for $f^{-1}(x)$ and $g^{-1}(x)$.

4. (**C6.1 - N**) (**CA**) For the following functions, determine the value of $f(2)$ as well as determining an expression for $f(2 + h)$, then use these values to determine the value of the difference quotient $\frac{f(2+h)-f(2)}{(2+h)-(2)}$:   **(Cirrito 18.3, p592)**

   a. The function $f(x) = x^3 + 2$.   b. The function $f(x) = \frac{1}{x+2}$

5. (**T3.4 - R**) (**CI**) The depth, $d(t)$ meters, of water at the entrance of a harbour at t hours after midnight on a particular day is given by $d(t) = 12 + 3\sin\left(\frac{\pi}{6} t\right)$, $0 \leq t \leq 24$.

   (Cirrito 10.5, p364)

   a. Sketch a graph of $y = d(t)$ for $0 \leq t \leq 24$
   b. For what value(s) of t will: (i) $d(t) = 10.5$, for $0 \leq t \leq 24$; (ii) $d(t) \geq 10.5$, for $0 \leq t \leq 24$

   Boats requiring a minimum depth of K meters are only permitted to enter the harbour when the depth of water is at least $K + 0.5$ meters for a continuous period of one hour.

   c. (**CA**) Find the largest value K, correct to 2 decimal places which satisfies the given condition.

Section B (Skills/Concepts Practice)

6. **(T3.5 - R) (CI)** SKILL: Linear Trigonometric Equations. Solve the following equations on the domain of $-180° \leq x \leq 180°$:

   a. $2\cos(x) - 1 = 0$ b. $2\sin(x) + \sqrt{3} = 0$ c. $\frac{1}{\sqrt{3}}\tan(x) + 1 = 0$

7. **(T3.5 - R) (CI)** SKILL: Linear Trigonometric Equations. Solve the following equations on the domain of $0 \leq x \leq 3\pi$:

   a. $\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) - 1 = 0$ b. $\sqrt{2}\sin(2x) + 1 = 0$

8. **(T3.5 - R) (CI)** SKILL: Quadratic Trig Equations. Factor the following expressions:

   a. (i) $1 - \cos^2 x$ (ii) $1 - 4\sin^2 x$ (iii) $\sin x - \sin^2 x$
   b. (i) $\sin^2 x - \cos^2 x$ (ii) $\cos^2 x + 2\cos x + 1$ (iii) $\sin^2 x - 2\sin x + 1$

9. **(T3.5 - R) (CI)** SKILL: Quadratic Trig Equations. Solve each of the following factored trigonometric equations given the domain of $0 \leq x \leq 2\pi$
   a. $\sin x \cos x = 0$
   b. $\sin x (\cos x - 1) = 0$
   c. $\cos x (\sin x + 1) = 0$

Section C (Skills/Concepts HW)

10. Quadratic Trig Equations; Oxford, Ex 13D, p456, Q3ab
11. Quadratic Trig Equations; Cirrito, Ex 10.4, p359, Q7a