

Math SL PROBLEM SET 30

Section A (Skills/Concepts Consolidation)

1. **(F2.3 - R) (CI)** Given the function $f(x) = \cos(x)$, $-2\pi \leq x \leq 2\pi$: **(Cirrito 6.1, 6.2; p167,177)**
a. Sketch f and label three (3) points on the graph.

The function $h(x)$ is defined as $h(x) = 4f\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right)$.

- b. State the domain and range, the x - and y -intercepts of $h(x)$ and sketch $h(x)$.
c. Evaluate $h^{-1}(-4)$.
2. **(A1.1 - R) (CI)** The sum of the first 8 terms of a geometric series is 17 times the sum of its first 4 terms. Find the common ratio. **(Cirrito 8.2.4, p264)**
3. **(T3.4 - R) (CI)** During the summer months, a reservoir supplies water to a city based upon the water demand modelled by the function $D(t) = 120 + 60 \sin\left(\frac{\pi}{90}t\right)$, $0 \leq t \leq 90$, where t measures the number of days from the start of summer (which lasts for 90 days). **(Cirrito 10.5, p361)**
a. Sketch the graph of $D(t)$.
b. What are the maximum and minimum demands made by the community over this period?
c. Evaluate and interpret $D(30)$.

However, the weather next summer will be significantly different than this year, so Mayor Santowski has modified the new water demand function to now be $N(t) = 1.5D(t) + 20$.

- d. Explain what this new function means and how it impacts your answers for the maximum and minimum demands for the next year.
4. **(A1.2, F2.6 - R) (CI)** Given the function $f(x) = \log_3(x)$ for $x > 0$; **(Cirrito 5.3.4, Cirrito 6.1, 6.2; p131,167,177)**
a. State the domain, range, asymptote(s) and intercepts for f and sketch this function.
b. **(CA)** Find the equation of the line that is tangent to $f(x)$ at $x = 5$. What is the significance of the slope of the tangent line?
c. Let $h(x)$ be the function $h(x) = 9 - x$. Determine the equations for $(f \circ h)(x)$ and $(h \circ f)(x)$ and sketch the 2 composite functions. Explain how the new functions relate to transformations of $f(x)$.
d. Let $k(x)$ be the function $k(x) = 9x$. Determine the equations for $(f \circ k)(x)$ and $(k \circ f)(x)$ and sketch the 2 composite functions. Explain how the new functions relate to transformations of $f(x)$.

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Section B (Skills/Concepts Practice)

5. **(A1.1 - E) (CI) SKILL:** Sigma Notation. Write an expression for each series using sigma notation. Then evaluate each sum.
- $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$
 - $9 + 16 + 25 + 36 + 49$
 - $27 + 25 + 23 + 21 + 19 + 17$
 - $240 + 120 + 60 + 30 + 15 + 7.5$
 - $5x + 6x + 7x + 8x + 9x + 10x$
 - $4 + 7 + 10 + 13 + \dots + 55$
6. **(A1.2 - R) (CI) SKILL:** Laws of Exponents. Simplify the following, leaving your final answer using only positive exponents.
- $\frac{(-2)^3 \times 2^{-3}}{(x^{-1})^2 \times x^3}$
 - $\frac{(-a)^3 \times a^{-4}}{(b^{-2})^{-2} b^{-5}}$
 - $\frac{(x-1)^{-3}}{(x+1)^{-1}(x^2-1)^2}$
 - $\frac{y(x^2)^{-1} + x^{-1}}{x+y}$
7. **(A1.2 - E) (CI) SKILL:** Laws of Logs. If $\log a = -5$, $\log b = 3$ and $\log c = 4$, evaluate each expression:
- $\log\left(\frac{ab^3}{100}\right)$
 - $\log\left(a^2 b \sqrt{c^3}\right)$
8. **(A1.2 - E) (CI) SKILL:** Laws of Logs: If $\log_3 x = K$, write each of the following in terms of K .
- $\log_3\left(\frac{x^5}{81}\right)$
 - $\log_3\left(\sqrt[4]{9x^8}\right)$
9. **(A1.3 - E) (CI) SKILL:** Binomial Expansion. Expand the following expressions:
- $(p + q)^6$
 - $(x - 2y)^6$
 - $(x^2 - 2y)^7$
 - $(2w^2 + \frac{1}{w})^7$

Section C (Skills/Concepts HW)

- Sigma Notation: Cirrito, Exercise 8.2.2, p260, Q4
- Laws of Exponents: Cirrito, Exercise 7.1.1, p200, Q1def,7acd
- Laws of Logarithms: Oxford, Exercise 4O, p126, Q3,5
- Binomial Expansion: Oxford, Exercise 6N, p187, Q6,7,8