# Math SL PROBLEM SET 21

## Section A (Short Answer)

1. (**SP5.4 - E**) (**CA**) The following data table shows the rainfall in cm in a city from 2000 to 2008. *(Oxford 10.3, p345; Oxford 10.4, p349)*

| year | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
|------|------|------|------|------|------|------|------|------|------|
| rainfall | 42 | 51 | 39 | 44 | 31 | 33 | 30 | 28 | 21 |

   a. Show this data on a scatterplot (on TI-84)
   b. Write a linear equation that can be used to model the relationship between the year and the amount of rain
   c. Determine the $r$-value and use it to describe the correlation.
   d. Predict the amount of rainfall for 2009

2. (**V4.1, V4.2 - N**) (**CA**) Point A has a position vector of $OA = -5i + 2j$ and point B has a position vector of $OB = 6i$ and point C has a position vector of $OC = 8i + 10j$. Draw a diagram showing these points and    *(Cirrito 12.4, p423; Cirrito 9.5.4, p300)*
   a. determine the triangle type for $\triangle ABC$.
   b. determine the the measure of angle CAB

3. (**T3.5 - E**) (**CI**) For the following trigonometric equations, start by (i) drawing the two special right triangles and (ii) drawing one cycle of a sine and a cosine curve.  *(Cirrito 10.4, p351)*
   a. Solve $\sqrt{3}\tan(x) + 1 = 0$ on the domain of $-2\pi \le x \le 2\pi$
   b. Solve $\sin(x) - 2\sin^2(x) = 0$ on the domain of $-360° \le x \le 360°$

4. (**F2.1 - E**) (**CI**) Given the functions $f(x) = x^2 - 4$ and let $g(x) = \sqrt{x+4}$. *(Cirrito 5.4.1, p148; Cirrito 5.4.2, p157)*
   a. State the domain and range of $f(x)$ and $g(x)$.
   b. Determine the equation of $f \circ g(x)$ as well as $g \circ f(x)$.
   c. What do you notice happening?
   d. Explain why this happens.

5. (**A1.1 - E**) (**CI**) In an arithmetic sequence, $u_6 = 3u_4$. Given that $u_8 = 50$, determine $u_1$ as well as the sum of the first 10 terms of this sequence.  *(Cirrito, 8.1, p241)*

6. (**A1.1 - N**) (**CA**) For the following series, write the first 5 terms of the series and then evaluate the requested sum.  *(Cirrito 8.1.3, p249)*
   a. $\sum_{n=1}^{8}(3n+1)$     b. $\sum_{n=1}^{5}(3^n - 2)$   c. Repeat using the summation command on the TI-84.

## Section B (Extended Response/Investigation)

7.  (**SP5.6, SP5.8 - R,N**) (**CA**) In a chess game, a player can either win, lose or draw. The probability that Vishi wins any game of chess is 0.6 and the probability that Vishi draws any game of chess is 0.3. Vishi plays two games of chess. The probabilities of winning, losing or drawing are independent. *(Cirrito 15.1.5, p506; Cirrito 14.2, p498; Cirrito 16.3, p544)*

    a.  Complete a tree diagram.
    b.  Find the probability that Vishi will win 2 games.
    c.  Find the probability that Vishi will not lose 2 games.
    d.  Show/explain that the probability of Vishi winning 3 out of 5 games is 0.3456.
    e.  Vishi plays 5 games. Prepare a table of values wherein you list (i) the number of games he wins and (ii) the probability of winning that number of games.

| # of games won | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| probability | | | | 0.3456 | | |

8.  (**CA**) Exponential Modeling word problem *(Cirrito 7.2, p209)*

    When a colony of wasps was studied, its population was found to be approximated by the model $P(t) = 50e^{0.1t}$, $t \geq 0$, where P is the population of wasps, and $t$ days is the time from the start of the study.
    (a)  i.  What was the population of the wasp colony when the study began?
        ii.  What was the population of the wasp colony 10 days after the study began?
    (b)  Sketch, on a set of axes, a graph of the population $P$ against the time $t$.
    (c)  In a sentence, describe how the population of the wasp colony was changing.

    Over the same period of time a second wasp colony was also studied. Its population, $Q$, was found to be approximated by the model $Q(t) = 500 - 450e^{-0.1t}$, $t \geq 0$.
    (d)  i.  What was the population of this second colony when the study began?
        ii.  What was its population 10 days after the study began?
    (e)  On the same set of axes as (b), sketch the graph of the population $Q$ against time.
    (f)  In a sentence, describe how the population of the wasp colony was changing.
    (g)  Using you graph, estimate the population when the populations of the two wasps colonies are the same.
    (h)  i.  By solving an appropriate equation, show that when the two wasp colonies have equal numbers, then $k^2 - 10k + 9 = 0$ where $k = e^{0.1t}$.
        ii.  Hence, find the exact time when the two wasp colonies have equal numbers.