Math SL PROBLEM SET 20

Section A (Short Answer)

1. **(F2.4, F2.7 - R) (CI)** For the following equations, state: (i) the number of solutions possible BEFORE you actually start solving them and (ii) then determine the solution(s) (if possible) *(Cirrito 2.4.2, p44, Oxford 2.3, p41)*

   a. $x^2 + 2x - 7 = 13 + x$
   b. $2k^2 + 11k = 5k - k^2 - 4$
   c. $2x - 1 = \frac{x+1}{2x}$

2. **(F2.2, F2.5 - R) (CI)** Given the function $f(x) = \frac{5}{x+1} + 2$, *(Cirrito 5.4.5, p144)*

   a. State the equation(s) of the asymptotes of f.
   b. Show that $\frac{5}{x+1} + 2$ is the same as $\frac{2x+7}{x+1}$.
   c. Hence, or otherwise, find the equation of $f^{-1}(x)$.

3. **(T3.5 - E,N) (CA)** Solve the following equations WITHOUT the use of graphs and systems ….. So in other words, present algebraic solutions to these questions (you may VERIFY your solutions using graphs and systems, however). The domain for all solutions is $0 \leq x \leq 2\pi$ (so all FINAL answers will be in radians) *(Cirrito 10.4, p351)*

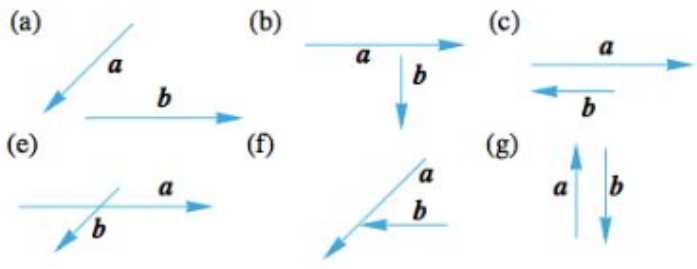
   a. $4 \sin x - 1 = 0$
   b. $(\cos x - 2)(3\cos x + 1) = 0$
   c. $\tan^2 x - 5 = 0$

4. **(T3.2, T3.5 - E,N) (CI)** Solve the following equations WITHOUT the use of your calculator - so in other words, special triangles. The domain for all solutions will be $0 \leq x \leq 2\pi$ (so all FINAL answers will be in radians) *(Cirrito 10.1.2, Cirrito 10.4, p351)*

   a. $2 \sin x - 1 = 0$
   b. $(\cos x - 1)(2\cos x + 1) = 0$
   c. $\tan^2 x - 3 = 0$

5. **(V4.1 - N) (CI)** For each of the following vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, use graph paper to draw: *(Cirrito 12.3, p415)*

   a. The resultant vector $\boldsymbol{c}$ which is $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$
   b. The resultant vector $\boldsymbol{d}$ which is $\boldsymbol{d} = \boldsymbol{a} - \boldsymbol{b}$



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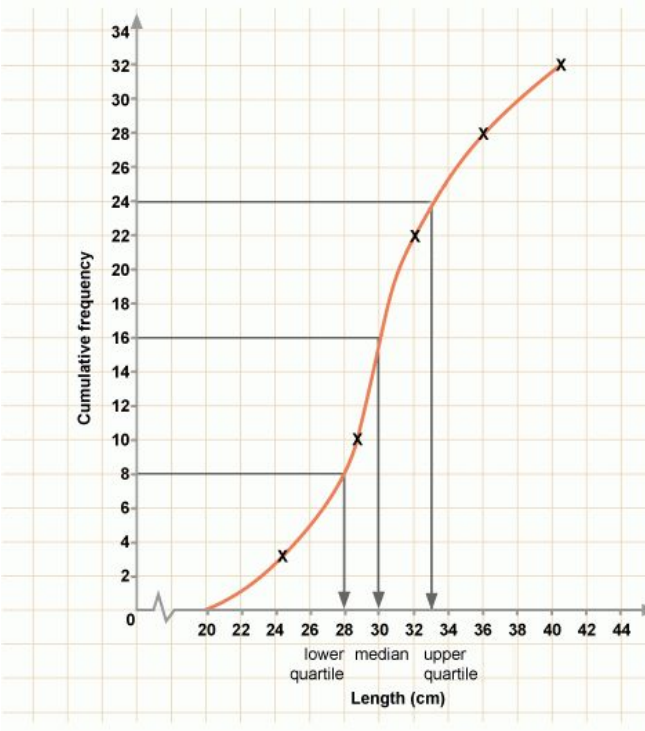
6. **(SP5.1, SP5.2 - R) (CA)** The following cumulative frequency graph (also known as ogives) shows the accumulated frequency of lengths of tails of cheetah cubs. ***(Oxford 8.5, p271)***



   a. Create a box and whisker plot from the GFG
   b. Create an appropriate frequency table from the CFG (HINT: think about a plausible interval width for your table)
   c. Hence, estimate the mean tail lengths.
   d. Draw a frequency histogram and a frequency polygon.

Section B (Extended Response/Investigation)

7. **(A1.1 - E,N) - (CA)** Two species of spiders inhabit a remote island. The population of Species A is 12,000 and is growing at a rate of 1.25% per month. The population of Species B is 50,000 and is decreasing at a rate of 175 spiders every month. ***(Oxford 6.8, p181)***

   a. Find the number of spiders of Species A in each of the first 6 months. What do you observe about this sequence of numbers?
   b. Find the number of spiders of Species B in each of the first 6 months. What do you observe about this sequence of numbers?
   c. When will the population of spiders from Species A first exceed that of Species B?

8. **(A1.2 - N) (CA)** Investigating the property of logarithms ***(Cirrito 7.4, p221)***

   a. Experiment with appropriate values to find $\log_a a$ and $\log_a 1$.
   b. Using your GDC, calculate the following:
      ln ½ ; ln 2 ; ln 3 ; ln 4 ; ln 5 ; ln 6 ; ln 7 ; ln 8 ; ln 9 ; ln 10

   c. Now, express each of the following in as many ways as possible using the values you have found. For example, ln4 = ln2 + ln2 = ln8 + ln½
      ln 24 = ; ln 25 = ; ln 40 = ; ln 72 = ; ln 100 =

   d. You have just discovered two very important laws of logarithms that can be generalized to any base: Write your conjectures:
      $\log_c(ab)$ = ?? ii. $\log_c a^k$ = ?? iii. Can you make a guess for $\log_c\left(\frac{a}{b}\right)$ = ???