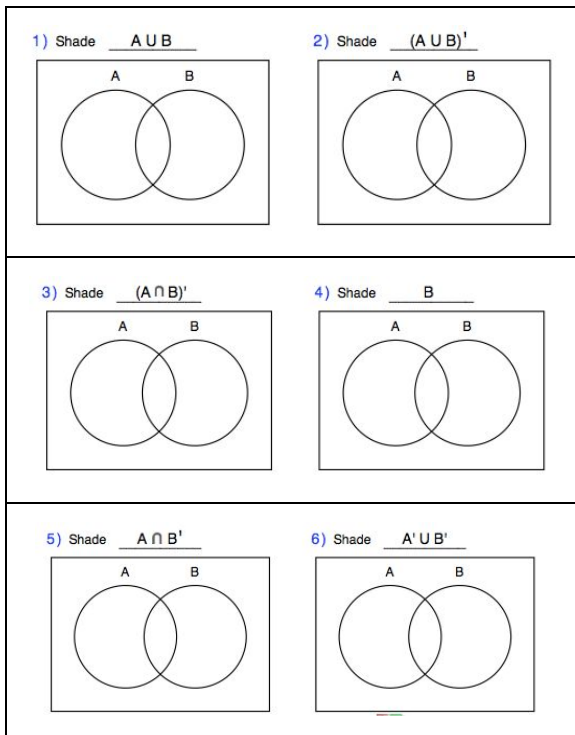# Math SL PROBLEM SET 15

## Section A (Short Answer)

1. **(SP5.5 - E) (CI)** Shade the following regions in the Venn diagrams: *(Cirrito 15.2, p508)*

   1) Shade ___A ∪ B___

   2) Shade ___(A ∪ B)'___

   3) Shade ___(A ∩ B)'___

   4) Shade ___B___

   5) Shade ___A ∩ B'___

   6) Shade ___A' ∪ B'___

2. **(F2.3 - E) (CI)** The graph of $y = f(x)$ is shown in the diagram. Using a different set of axes for each graph, sketch the graphs of each of the following transformed functions, $g(x)$, showing clearly any intercepts or extrema (maximums or minimums) in the "new" functions. *(Cirrito 6.1, p167; 6.2, p177; 6.3, p183)*

   a. $g(x) = \frac{2}{3} f(x)$

   b. $g(x) = -f(-x)$
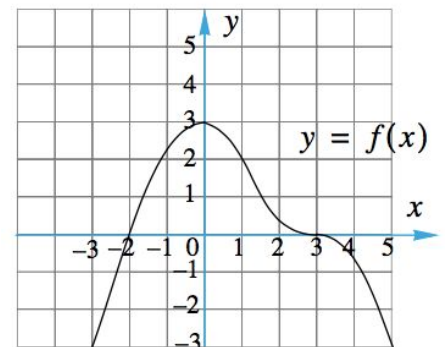
   c. $g(x) = f(x + 2) + 1$

3. **(V4.1 - N) (CI)** Vector Addition. Go to the following geogebra animation to find out how to add two vectors. Explain the strategy being used. https://www.geogebra.org/m/HMje3hdt *(Cirrito 12.3, p415)*

4. **(V4.1 - N) (CI)** Vector Subtraction: Go to the following geogebra animation to find out how to subtract two vectors. Explain the strategy being used: https://www.geogebra.org/m/vUAFWvmk *(Cirrito 12.3, p415)*

5. **(SP5.1, SP5.2, SP5.3 - R) (CA)** A group of 100 IB students was given a math test that was graded out of 20 points. The following table shows the distribution of the marks obtained: *(Cirrito 13.2, p471)*

| mark | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|------|---|----|----|----|----|----|----|----|----|----|----|
| number of students | 1 | 1 | 3 | 5 | 8 | 13 | 19 | 24 | 14 | 10 | 2 |

   a. Write down the mode.
   b. Draw a cumulative frequency graph.
   c. Calculate the mean.
   d. Find the median.
   e. Find the upper and lower quartiles
   f. Draw a box & whisker plot for the data.
   g. Another group of 50 students had a mean mark of 17.16 on the same test. Calculate the mean of the entire group of 150 students.

6. **(A1.3, SP5.8 - N) (CA)** Given the binomial expression of $(a + b)^7$: *(Cirrito 4.1, p95)*

   a. When expanded, the expression is: $(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)(a + b)$. Use this idea to explain **why** the coefficient of the $a^3b^4$ term is 7 ***nCr*** 3 (or $_7C_3$ or $\binom{7}{3}$ )

   b. If $a$ represents the probability of selecting a CAC student with a US passport and is equal to 0.42, determine the probability of randomly selecting 3 students with a US passport from a group of 7 CAC students.

   c. Find the coefficient of the $x^2$ term in the expansion of $(2x - 1)^7$.

### Section B (Extended Response/Investigation)

7. **(F2.6 - R) (CA)** Investigating Compound interest and **the natural base**, e. *(Cirrito 7.1.5, p207)*

   a. Determine the future value of a $10,000 investment that has been earning 5% p.a. compounded monthly for 6 years.

   b. A population of fish in the Nile River is modelled by the equation $P(t) = 120e^{0.075\,t}$, where $t$ is time in years since January 1st of the year 2000 and $P(t)$ is in thousands.

      i. How many fish are there in the Nile today (end of Oct of 2018)?
      ii. When will the population of fish be 720,000?

   c. Complete the following "investigation" and explain what the idea of continuous compounding means? Where do we get the base "e" from???

GIVEN: the formula for working with compound interest ➜ $FV = PV\left(1+\dfrac{i}{n}\right)^{nt}$, determine the value after 1 year of a \$1 investment invested at 100% pa under the following compounding conditions: **(NOTE: set up on TI-84 as well)**

| | |
|---|---|
| (a) 100% pa compounded annually | |
| (b) 100% pa compounded semi-annually | |
| (c) 100% pa compounded quarterly | |
| (d) 100% pa compounded monthly | |
| (e) 100% pa compounded daily | |
| (f) 100% pa compounded hourly | |
| (g) 100% pa compounded every minute | |
| (h) 100% pa compounded every second | |
| (i) 100% pa compounded n times per year | |

FINAL QUESTION? ➜ BY WHAT **RATIO** HAS YOUR MONEY INCREASED IN VALUE?

8. **(V4.1 - N) (CI)** The two previous questions have shown you how to work with **algebraic** vectors. Explain what algebraic vectors are. Then, add/subtract the following vectors, as required: ***(Cirrito 12.3, p415)***

   a. Given that $\vec{a} = 2\vec{i} + 6\vec{j}$ and $\vec{b} = -4\vec{i} + 8\vec{j}$, find and illustrate with a diagram:

   i. $\vec{a} + \vec{b}$        ii. $\vec{a} - \vec{b}$     iii. $\vec{b} - \vec{a}$     iv. $\vec{b} + \vec{a}$

   ii. $5\vec{a}$          vi. $5\vec{a} - 2\vec{b}$

   b. The vectors $p$ and $q$ are defined below as 3D vectors:

   The vectors $p$ and $q$ are defined by $p = \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$ and $q = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}$. Find:

   (i)    $p + 2q$      (ii)    $-3p - 5q$    (iii)    $3p$    (iv)    $2p + 3q$