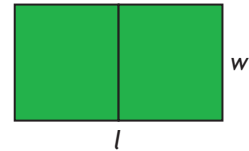# Different Forms of the Equation of a Line

GOAL

Given an equation in the form $Ax + By + C = 0$ or $Ax + By = D$, express the equation in the form $y = mx + b$.

LEARN ABOUT the Math

David is a dog breeder and needs to construct two identical, adjacent rectangular pens to contain the male and female puppies. He has 24 m of fencing material available.
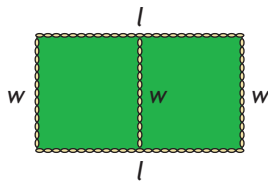
? What are some possible values for the length and width of the pens?

EXAMPLE 1 | Using an algebraic strategy

Determine possible lengths and widths for the pens.

Pietr's Solution: Isolating *l* and using the slope and y-intercept to sketch a graph

l represents the length of the pens and *w* represents the width.

$$2l + 3w = 24$$

> I chose variables to represent each value.
>
> I wrote an equation using two lengths and three widths to represent the total amount of fencing.

$$2l + 3w - 3w = 24 - 3w$$
$$2l = -3w + 24$$
$$\frac{2l}{2} = \frac{-3w + 24}{2}$$
$$\frac{2l}{2} = \frac{-3w}{2} + \frac{24}{2}$$
$$l = -\frac{3}{2}w + 12$$
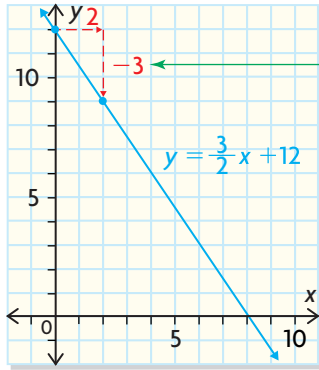$$y = -\frac{3}{2}x + 12$$

> I used inverse operations to isolate the length variable *l*, so that I could calculate a length to go with any width I chose.
>
> I replaced *l* with *y* since it is the dependent variable and *w* with *x* since it is the independent variable.

$$y = -\frac{3}{2}x + 12$$

The equation is in the form $y = mx + b$.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = -\frac{3}{2}$$

$$y\text{-intercept} = 12$$

I know that m tells the slope and b tells the y-intercept.

I plotted the y-intercept first.

Starting at the y-intercept, I used the run to move 2 units right and the rise to move 3 units down to determine a second point on the line.

Then, I drew the line joining the two points.

Some possible dimensions for the pens are:

$w = 2$ m and $l = 9$ m

$w = 3$ m and $l = 7.5$ m

$w = 4$ m and $l = 6$ m

$w = 6$ m and $l = 3$ m

I used the graph to locate other points that were on the line. Each ordered pair (x, y) or (w, l) represents the dimensions of a pen with a perimeter of 24 m.

Hanxiang's Solution: Isolating a variable to graph the relation with a graphing calculator

l represents the length of the pens and w represents the width.

I chose variables to represent each value.

The diagram has 2 horizontal and 3 vertical sides, so I wrote a sum equal to the amount of fencing.

$$2l + 3w = 24$$

$$2l + 3w - 2l = 24 - 2l$$

$$3w = -2l + 24$$

$$\frac{3w}{3} = \frac{-2l + 24}{3}$$

$$\frac{3w}{3} = \frac{-2l}{3} + \frac{24}{3}$$

$$w = -\frac{2}{3}l + 8$$

My graphing calculator requires linear relations to be entered in the form $y = mx + b$. I decided to isolate w so that l would be the independent variable.

I entered the equation into the graphing calculator. I replaced the independent variable *l* with *x*, and the dependent variable *w* with *y*.
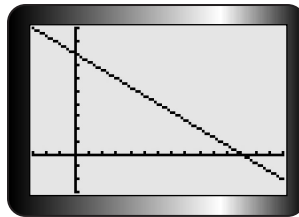
Since I knew that both the width and the length had to be positive, I changed my window settings so that I could focus on the graph's values in quadrant 1.
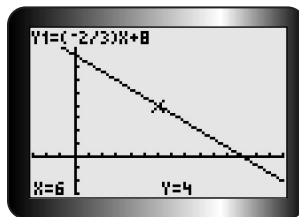
$l = 6$ and $w = 4$

I used the value operation to get some possible values for *l* and *w*.
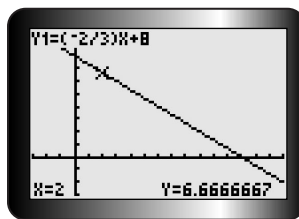
$l = 2$ and $w = 6.\overline{66}$

$l = 9$ and $w = 2$

Some possible dimensions are:
$l = 6$ m and $w = 4$ m
$l = 9$ m and $w = 2$ m
$l = 2$ m and $w \doteq 6.7$ m

This used a little more than 24 m of fencing, so I knew that the answer wasn't exact.

Reflecting

A. Why do Pietr's and Hanxiang's forms of the equation give some of the same values for l and w?

B. How did isolating a variable help each student to solve the problem?

APPLY the Math

EXAMPLE 2	Using an algebraic strategy to determine the slope and the y-intercept

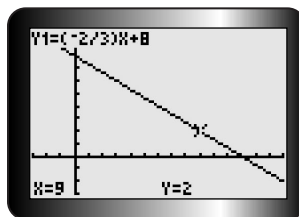Determine the slope and the y-intercept of the line $3x + 4y + 8 = 0$.

Sara's Solution

$$3x + 4y + 8 = 0$$
$$3x - 3x + 4y + 8 - 8 = 0 - 3x - 8$$
$$4y = -3x - 8$$
$$\frac{4y}{4} = \frac{-3x - 8}{4}$$
$$y = \frac{-3}{4}x - \frac{8}{4}$$
$$y = \frac{-3}{4}x - 2$$

I wanted the equation in the form $y = mx + b$, to determine the values of m and b.

I used inverse operations to isolate y.

I knew that if the equation was in the form $y = mx + b$, m would give the slope, and b would give the y-intercept.

The slope is $-\dfrac{3}{4}$ and the y-intercept is -2.

EXAMPLE 3	Using an equation to represent and solve a problem

Sam has two part-time jobs. At the grocery store he earns \$8/h and at the library he earns \$10/h. Before going on vacation, he would like to save \$280. Determine the fewest number of hours he needs to work to achieve his goal.

Aaron's Solution

G is the number of hours worked at the grocery store. L is the number of hours worked at the library.

I chose variables for the two unknowns.

$$8G + 10L = 280$$

$$8G - 8G + 10L = -8G + 280$$

$$10L = -8G + 280$$

$$\frac{10L}{10} = \frac{-8G + 280}{10}$$

$$\frac{10L}{10} = \frac{-8G}{10} + \frac{280}{10}$$

$$L = \frac{-4}{5}G + 28$$

I had to multiply each hourly rate by the number of hours to get the total earnings.

I used inverse operations to isolate L.

G	L	Total Hours Worked
0	$-\frac{4}{5}(0) + 28$ $= 28$	28
5	$-\frac{4}{5}(5) + 28$ $= 24$	29
7.5	$-\frac{4}{5}(7.5) + 28$ $= 22$	29.5
20	$-\frac{4}{5}(20) + 28$ $= 12$	32
45	$-\frac{4}{5}(45) + 28$ $= -8$	?

I calculated some possible solutions for the problem by choosing a value for G and substituting it into my equation.

My last choice meant that Sam worked a negative number of hours at the library, which is impossible. On the other hand, if Sam worked 45 hours at the grocery store he would earn $360, which is more than the $280 he wants to save.

If Sam worked 0 h at the grocery store and 28 h at the library, he would earn enough money for his vacation.

I chose the answer that showed the fewest total hours Sam had to work to earn $280.

I knew that this made sense because (28 h)($10/h) = $280.

In Summary

Key Idea

- You can take an equation that is in the form $Ax + By + C = 0$ or $Ax + By = D$ and rewrite it into the form $y = mx + b$ by using inverse operations to solve for y.
- You can locate two points on most lines by plotting the y-intercept and locating a second point using the rise and run of the slope. Joining these points with a straight line gives you a sketch of the relation.

Need to Know

- Equations in the form $Ax + By + C = 0$, $Ax + By = D$, and $y = mx + b$ represent linear relations.
- To enter equations into a graphing calculator, write linear equations in the form $y = mx + b$.

CHECK *Your Understanding*

1. Express the equation $5x + 6y + 15 = 0$ in the form $y = mx + b$.

2. A room contains three-legged stools and four-legged chairs. There are 48 legs altogether.
   a) Write an equation to represent the relationship between the number of stools, the number of chairs, and the total number of legs.
   b) How many stools could there be?

PRACTISING

3. Express each of the following equations in the form $y = mx + b$.
K Then, state the slope and y-intercept of each line.
   a) $4x - 3y = 24$
   b) $2x + 5y = 15$
   c) $3x - 6y - 14 = 0$
   d) $8x + 5y = 0$
   e) $4x + 7y - 11 = 0$
   f) $2.4x + 1.5y = -3$

4. Use the slope and y-intercept to sketch the graphs of each of the linear relations in question 3.

5. a) Without graphing, predict whether each of the following lines will rise or fall to the right. How do you know?
   i) $2x + 3y = 5$
   ii) $x - 4y + 10 = 0$
   iii) $3x + 5y - 8 = 0$
   iv) $2x + 5y = 15$
   v) $2.5x - 15y = 20$
   vi) $\dfrac{x}{2} - 3y = 6$

   b) Check your predictions by graphing each line.

6. The dependent variable is d in each of the following equations. Isolate d to determine the d-intercept and the slope of each line.

   a) $4t + 3d = 9$

   b) $8d - 2h + 16 = 0$

   c) $15 + 5k - 6d = 0$

7. A farmer wants to build new enclosures for geese, ducks, and chickens.

A He has 40 m of fencing to build the three identical, adjacent enclosures.

   a) Write an equation to represent the amount of fencing required.

   b) Rearrange your equation to isolate one of the variables.

   c) Graph the relationship.

   d) Suggest three possible sets of dimensions for the farmer's enclosures.

8. Evan spent a total of $18 on gourmet jellybeans and chocolate-covered almonds. The jellybeans cost $12/kg. The almonds cost $21/kg.

   a) Write an equation to represent Evan's purchases.

   b) Isolate the variable for the quantity of jellybeans in your equation.

   c) If Evan bought 250 g of almonds, how many grams of jellybeans did he buy?

   d) If Evan bought 100 g of almonds, how many grams of jellybeans did he buy?

9. Orenda has a total of 41 loonies and toonies in her piggy bank. Their

T total value is $59.

   a) Write one equation for the total number of coins and a second equation for the total value.

   b) Graph both lines.

   c) Determine the coordinates of the point of intersection of the lines.

   d) How do you know that the coordinates of the point of intersection are the only possible combination of loonies and toonies that meets the conditions of this situation?

10. Amanda plans to make chocolate-chip cookies
and oatmeal cookies for a bake sale. The
chocolate-chip cookies use three eggs per
batch. The oatmeal cookies use two eggs per
batch. How many batches of each recipe can
she make using two dozen eggs?

11. Textbooks have an average mass of 0.9 kg and
notebooks have an average mass of 0.6 kg. To avoid straining his back,
Stephen never puts more than 6 kg of books in his backpack.
   a) Write an equation to represent the relationship between the
   number of each type of book and the total maximum mass.
   b) Isolate one of the variables in your equation from part a).
   c) Determine all possible combinations of textbooks and notebooks
   that would have a total mass of 6 kg.

12. a) Show that $3x - 8y + 5 = 0$ and $y = \dfrac{3}{8}x + \dfrac{5}{8}$ represent the
   same line.

   b) Do $y = \dfrac{2}{3}x + \dfrac{1}{3}$ and $2x + 3y + 1 = 0$ represent the same line?
   How do you know?

13. Punitha really only understands how to graph a line if it is in the form
   C $y = mx + b$.
   a) As her tutor, how would you ensure that Punitha is able to graph
   lines expressed in any form?
   b) What can you tell Punitha about the similarities between all the
   linear equation forms?

Extending

14. Show that $y = \dfrac{2}{3}x + \dfrac{7}{3}$ and $x = \dfrac{3}{2}y - \dfrac{7}{2}$ represent the same line.

15. a) Determine the slope and y-intercept for each linear equation.
   i) $3x + 4y - 8 = 0$
   ii) $2x + 5y - 9 = 0$
   iii) $4x - 3y = -12$
   b) An equation is given in the form $Ax + By + C = 0$.
   i) What is the slope of this line?
   ii) What is the y-intercept of this line?