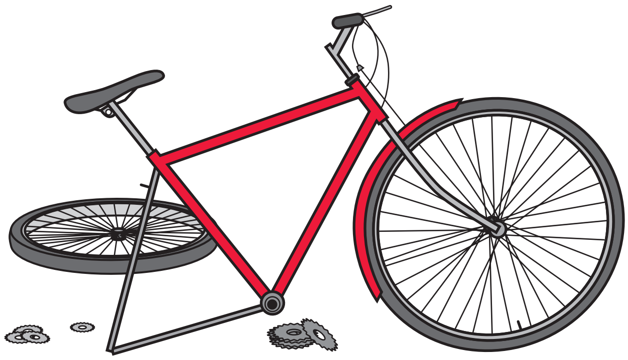
4.4 Solving for a Variable in a Linear Relation

GOAL

Use inverse operations to solve for a variable in a linear relation.

LEARN ABOUT the Math

Ralph and Bill work part time repairing bikes. They are paid \$2 to install a tire and \$5 to install gears. Their boss will pay a maximum of \$100 per week.



? Which combinations of tire and gear installations will earn the boys exactly \$100?

EXAMPLE 1 Strategies for determining ordered pairs in a linear relation

The boys developed the linear relation $2T + 5G = 100$ to represent the situation. T represents the number of tire installations and G represents the the number of gear installations.

Determine the combinations of tire and gear installations that will earn them \$100.

Ralph's Solution: Using guess-and-check to solve for one variable after substituting a value for the other

Let $T = 5$. So, $2T + 5G = 100$.	I chose a value for T. I knew the value of G should depend on the value of T that I chose.

$$2(5) + 5G = 100$$
$$10 + 5G = 100$$
$$10 + 5G - 10 = 100 - 10$$
$$5G = 90$$
$$\frac{5G}{5} = \frac{90}{5}$$
$$G = 18$$

I substituted the value for T into the equation and solve for G.

Let $T = 6$:

I chose a different value for T.

$$2T + 5G = 100$$
$$2(6) + 5G = 100$$

I substituted the value into the equation.

$$12 + 5G - 12 = 100 - 12$$
$$5G = 88$$
$$\frac{5G}{5} = \frac{88}{5}$$

I used inverse operations to solve for G.

$$G = 17.6$$

It is impossible to do 17.6 gear installations, so $T = 6$ is not a reasonable value to choose.

Each time I chose a value, I ended up with an equation like $5G = \blacksquare$. I realized that whatever I subtract from 100 must be a multiple of 5 in order to divide and get a whole number. T must be a multiple of 5.

I tried $T = 10$ as a value.

Let $T = 10$:

$$2T + 5G = 100$$
$$2(10) + 5G = 100$$
$$20 + 5G - 20 = 100 - 20$$
$$5G = 80$$
$$\frac{5G}{5} = \frac{80}{5}$$
$$G = 16$$

5 tire installations and 18 gear installations will earn exactly \$100. So will 10 tire installations and 16 gear installations.

Bill's Solution: Using algebra to rearrange the relation to solve for one variable in terms of the other

$$2T + 5G - 5G = 100 - 5G$$
$$2T = 100 - 5G$$

I thought I could save time if I wrote an equivalent relation that showed how to calculate one of the variables in terms of the other.

I decided to write an equation that **solved for T in terms of G**. That meant I first had to undo the $+ 5G$ on the left.

solve for a variable in terms of other variables

the process of using inverse operations to express one variable in terms of the other variable(s)

$$\frac{2T}{2} = \frac{100 - 5G}{2}$$
$$T = \frac{100 - 5G}{2}$$

I used inverse operations to isolate the variable T.

Let $G = 10$:

$$T = \frac{100 - 5G}{2}$$
$$T = \frac{100 - 5(10)}{2}$$
$$T = \frac{50}{2}$$
$$T = 25$$

I chose a value for G and substituted the value into the equation.

10 gear installations and 25 tire installations will earn exactly $100.

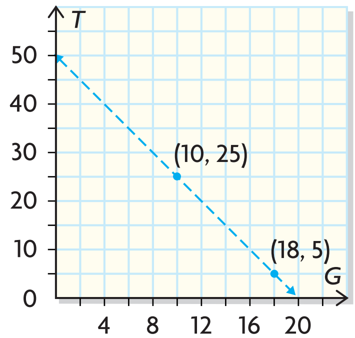
Let $G = 11$:

$$T = \frac{100 - 5G}{2}$$
$$T = \frac{100 - 5(11)}{2}$$
$$T = \frac{45}{2}$$
$$T = 22.5$$

I chose a different value for G and substituted the value into the equation.

It is impossible to do 22.5 tire installations, so $G = 11$ is not a reasonable value to choose.

$$T = \frac{100 - 5G}{2}$$



I also found that 18 gear and 15 tire installations would work, so I plotted the points (10, 25) and (18, 5) and joined these with a dashed line since that data is discrete. This gave me the graph of the relation

$$T = \frac{100 - 5G}{2}.$$

Points outside the first quadrant are not possible because they would mean a negative number of tire or gear installations.

I can see from the graph that there are several combinations that would earn exactly \$100. Some possible combinations are (0, 50), (2, 45), (4, 40), (6, 35), (8, 30).

I noticed that each increase of 2 tire installations resulted in a decrease of 5 gear installations.

Any point that lies on the line that has whole-number coordinates represents a combination that will earn \$100.

Reflecting

A. How did Ralph and Bill know which inverse operations to use and in what order to apply them?

B. How would Bill's solution, equation, and graph be different if he chose to solve for G instead of T?

C. How is using inverse operations to solve a linear equation similar to using inverse operations to solve for one variable in terms of another? How is it different?

APPLY the Math

EXAMPLE 2 Using an inverse operations strategy to solve a relation for one variable in terms of the other

Solve for y in terms of x for the line $\frac{2}{3}x + \frac{1}{5}y = 2$.

Agatha's Solution

$$\frac{2}{3}x + \frac{1}{5}y - \frac{2}{3}x = 2 - \frac{2}{3}x$$

I used an inverse operation to remove the $\frac{2}{3}x$ term and isolate the $\frac{1}{5}y$ term on the left.

$$\frac{1}{5}y = 2 - \frac{2}{3}x$$

$$5\left(\frac{1}{5}y\right) = 5\left(2 - \frac{2}{3}x\right)$$

Then, I multiplied both sides by 5 to solve for y. I added brackets to remind me to use the distributive property on the right side of the equation.

$$y = 10 - \frac{10}{3}x$$

I simplified the right side of the equation.

$$y = -\frac{10}{3}x + 10$$

I reordered the terms on the left.

The equation $\frac{2}{3}x + \frac{1}{5}y = 2$ is the same as $y = -\frac{10}{3}x + 10$.

EXAMPLE 3 Rearranging an equation before solving it

Vicki puts $1500 in an investment account that earns 7.5% simple interest per year. If she wants to earn $200 in interest, how long must she leave the money in the investment?

Latifa's Solution

The simple interest formula is: $I = prt$.
- I represents the interest earned;
- p represents the principal amount invested (the initial amount put into the bank account);
- r represents the interest rate per year (expressed as a decimal);
- t represents the time (in years) that the money was invested.

$$I = prt$$

$$\frac{I}{pr} = \frac{prt}{pr}$$

$$\frac{I}{pr} = t$$

Since I wanted to calculate the time needed for the investment, I decided to solve for t. I had to undo the multiplication by p and by r. The inverse of multiplying by p and r is to divide by p and r.

$$I = \$200$$
$$r = 0.075$$
$$p = \$1500$$
$$\frac{200}{1500 \times 0.075} = t$$

$r = 7.5\% = \frac{7.5}{100} = 0.075$
I substituted the values for I, p, and r into the equation for t.

$$1.8 = t$$

1.8 years is about 1 year and 80% of the second year.

$$80\% \text{ of } 12 \text{ months}$$
$$= \frac{80}{100} \times 12$$
$$= 0.80 \times 12$$
$$\doteq 10 \text{ months}$$

I calculated the number of months.

Vicki must leave her money in the investment for about 1 year and 10 months to earn $200 in interest.

In Summary

Key Idea

- You can use inverse operations to isolate any variable in a relation. This has the effect of solving for that variable in terms of the other variable(s) in the relation.

Need To Know

- The following strategy can be used to solve a relation for any variable:
  - Imagine that each of the other variables has been replaced by a number.
  - List the inverse operations needed to solve for the target variable.
  - Perform the inverse operations, one at a time, on the original relation, using the original variables until the target variable is isolated.

CHECK Your Understanding

1. Solve for the variable indicated.

a) $3x + y = 5$; solve for x

b) $2x + 5y = -10$; solve for y

2. Phil sharpens skates at a local sporting goods store. The store charges \$5.00 to sharpen a pair of figure skates and \$4.00 for a pair of hockey skates. On Saturday the store earned \$228 through skate sharpening.

   a) To model this situation, Phil wrote: \$5(number of pairs of figure skates) + \$4(number of pairs of hockey skates) = \$228. Write the relation using variables.
   b) Write the equation that expresses the number of hockey skates sharpened in terms of the number of figure skates.
   c) Write the equation that expresses the number of figure skates sharpened in terms of the number of hockey skates.

Practising

3. In each set of equations, identify the equation that is *not* equivalent to the others.
   a) $2a - b = 4$; $2a = b + 4$; $a = \frac{b}{2} + 2$; and $b = 2a + 4$
   b) $x + 2y = -6$; $y = \frac{x}{2} + 3$; $x = 2y - 6$; and $x - 2y + 6 = 0$
   c) $4m - 3n + 2 = 4$; $3n = 4m + 2$; $4m = 3n - 2$; and $3n - 4m = 2$

4. Solve for y in terms of x.
   K a) $2y = 8 - 4x$
   b) $-2x - 3y = 12$
   c) $2.8x + 1.1y - 5.3 = 0$
   d) $\frac{7}{5}y + \frac{2}{3}x = \frac{11}{13}$
   e) $\frac{4}{5} = \frac{2}{3}x + 1\frac{1}{2}y$
   f) $3(y - 2) + 2x = 8$

5. A cell-phone company offers a plan of \$25 per month and \$0.10 per minute of talk. The cost, C, in dollars, is given by the relation $C = 25 + 0.10n$, where n is the number of minutes used per month. Each month the company uses the exact air time to calculate the monthly bill.

a) Solve the relation for n in terms of C.
b) Create a table of values for this new relation.
c) Graph this relation.
d) What is the independent variable? What is the dependent variable?
e) Why might someone want to rearrange this relation and express it in terms of the cost?

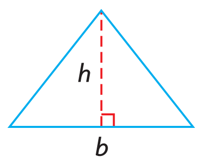
6. Start with the relation $2x - 5y = 20$.

a) Solve for y in terms of x.
b) Graph this relation using x as the independent variable.
c) State the slope and the intercepts of the graph.
d) Solve for x in terms of y.
e) Graph the relation using y as the independent variable.
f) State the slope and intercepts of the graph.
g) Compare the slope of the two graphs. Justify your comparison.

7. Solve the relation or formula for the variable indicated:

a) $2a - 5b = 12$; solve for a
b) $0.35m + 2.4n = 9$; solve for n
c) $\frac{1}{2}p - \frac{2}{3}q = \frac{1}{4}$; solve for p
d) $I = prt$; solve for r
e) $P = 2L + 2W$; solve for L
f) $C = 2\pi r$; solve for r

8. Look at the diagram.



T **a)** Write a formula for h in terms of the base, b, and the area, A.
b) Determine the height of the triangle if the area is 55 cm^2 and the base is 4 cm.

9. Ben has \$42.50 in quarters and dimes.

a) Write a linear relation expressing the total amount of money in terms of the number of quarters and dimes.
b) Write an equation to express the number of quarters in terms of the number of dimes.
c) Write an equation to express the number of dimes in terms of the number of quarters.
d) Use one of your equations to determine the possible combinations of quarters and dimes Ben could have.

10. A candy store is making a mixture of chocolate-coated almonds and
A chocolate-coated raisins. The almonds cost \$30/kg and the raisins cost \$8/kg. The total cost of the mixture is to be \$150.

a) Write a linear relation expressing the total cost in terms of the mass of almonds and the mass of raisins purchased.
b) Write an equation to express the mass of almonds in terms of the mass of raisins.
c) Write an equation to express the mass of raisins in terms of the mass of almonds.
d) Which combinations of almonds and raisins will cost exactly \$150?

11. The Alltime Watch Company makes and sells two kinds of watches.
C The profit on digital watches is \$15 per watch. The profit on analog watches is \$20 per watch. The watch factory can only produce watches in the ratio of 3 digital : 2 analog because of the machines it uses. Given this ratio, how many watches of each type must be produced to meet the company's profit target of at least \$20 000 per week?

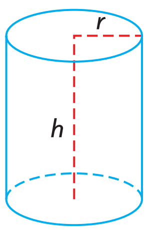
12. When you multiply a number, x, by k, add n, and then divide by r, the answer is w.

a) Write the relation that models this situation.
b) List the inverse operations that you would use, in the correct order, to isolate x.
c) Solve the relation for x.
d) How is rearranging a relation or formula for a particular variable similar to isolating a variable in a linear equation? How is it different?

Extending

13. Solve for x.

a) $\frac{5}{x} + 2y = 9$
b) $3x^2 + 50 = 197$
c) $(x - 4)^2 - 12 = 24$
d) $\frac{(3 + y)}{x} = -4$
e) $\sqrt{x + 1} = 9$
f) $2 - 8x^3 = 3$



14. The formula for determining the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$.

a) Solve for h in terms of SA and r.
b) Determine the height of a cylinder with radius 5 cm and surface area 300 cm^2.
c) Solve for r in terms of the other variables.