

1.1

Representing Linear Relations

YOU WILL NEED

- grid paper
- ruler
- graphing calculator



GOAL

Use tables, graphs, and equations to represent linear relations.

LEARN ABOUT the Math

Aiko's cell-phone plan is shown here. Aiko has a budget of \$30 each month for her cell phone.

| Services | Cost |
|---------------|-------------|
| calls | 20¢/min |
| text messages | 15¢/message |

- ❓ How can Aiko show how many messages and calls she can make each month for \$30?

EXAMPLE 1 Representing a linear relation

Show the combinations of messages and calls that are possible each month for \$30.

Aiko's Solution: Using a table

| Text Messages | | Calls | | Total Cost (\$) |
|--------------------|-----------|-------------------|-----------|-----------------|
| Number of Messages | Cost (\$) | Number of Minutes | Cost (\$) | |
| 0 | 0 | 150 | 30 | 30 |
| 20 | 3 | 135 | 27 | 30 |
| 40 | 6 | 120 | 24 | 30 |
| : | | : | | : |
| 200 | 30 | 0 | 0 | 30 |

I made a table to show how many messages and calls are possible for \$30. I started with 0 messages and let the number of messages increase by 20 each time. I calculated the cost of the messages by multiplying the number in the first column by \$0.15. Then I subtracted the cost of the messages from \$30 to determine the amount of money that was left for calls. I calculated the number of minutes for calls by dividing this amount by \$0.20.

As the number of text messages increases, the number of minutes available for calls decreases. Aiko can make choices based on the numbers in the table. For example, if Aiko sends 40 text messages, she can talk for 120 min.

40 text messages a month is about 1 per day. 120 min a month for calls is about 4 min per day.



Malcolm's Solution: Using an equation and a graph

Let x represent the number of text messages per month. Let y represent the number of minutes of calls per month.

I used letters for the variables.

Aiko has a budget of \$30 for text messages and calls, so $0.15x + 0.20y = 30$.

I wrote an equation based on Aiko's budget of \$30. In my equation, x text messages cost $\$0.15x$ and y minutes of calls cost $\$0.20y$.

At the x -intercept, $y = 0$.

$$0.15x + 0.20(0) = 30$$

$$x = \frac{30}{0.15}$$

$$x = 200$$

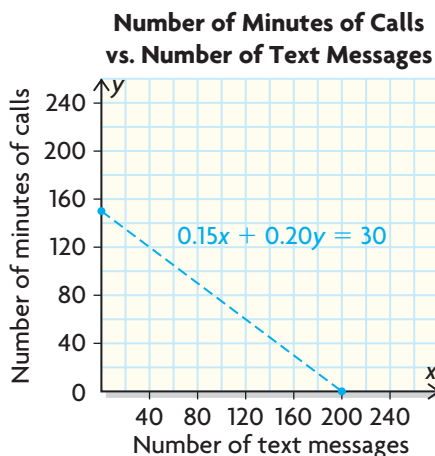
At the y -intercept, $x = 0$.

$$0.15(0) + 0.20y = 30$$

$$y = \frac{30}{0.20}$$

$$y = 150$$

I used my equation to calculate the maximum number of text messages and the maximum time for calls. To do this, I determined the intercepts.



I drew a graph by plotting the x -intercept and y -intercept, and joining them.

I used a broken line because x represents whole numbers only in this equation.

The point $(40, 120)$ shows that if Aiko sends 40 text messages in a month, she has a maximum of 120 min for calls to stay within her budget.

Aiko's options for text messages and calls are displayed as points on the graph. Each point on the graph represents an ordered pair (x, y) , where x is the number of text messages per month and y is the number of minutes of calls per month.

Reflecting

- How does the table show that the relationship between the number of text messages and the number of minutes of calls is linear?
- How did Malcolm use his equation to draw a graph of Aiko's choices?
- Which representation do you think Aiko would find more useful: the table or the graph? Why?

APPLY the Math

EXAMPLE 2

Representing a linear relation using graphing technology



Career Connection

Careers as diverse as sales consultants, software developers, and financial analysts have roles in currency exchange.

Patrick has saved \$600 to buy British pounds and euros for a school trip to Europe. On the day that he goes to buy the currency, one pound costs \$2 and one euro costs \$1.50.

- Create a table, an equation, and a graph to show how many pounds and euros Patrick can buy.
- Explain why the relationship between pounds and euros is linear.
- Describe how Patrick can use each representation to decide how much of each currency he can buy.

Brittany's Solution

- Let x represent the pounds that Patrick buys. Let y represent the euros that he buys.

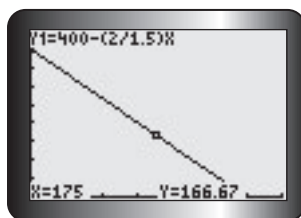
I chose letters for the variables. x pounds cost $\$2x$ and y euros cost $\$1.50y$. Patrick has \$600.

$$\begin{aligned}
 2x + 1.50y &= 600 \\
 1.50y &= 600 - 2x \\
 \frac{1.50y}{1.50} &= \frac{600}{1.50} - \frac{2x}{1.50} \\
 y &= 400 - \left(\frac{2}{1.50}\right)x
 \end{aligned}$$

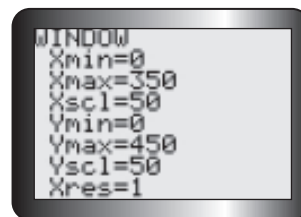
I wrote an equation based on the cost of the currency. I rearranged my equation into the form $y = mx + b$ so I could enter it into a graphing calculator.

Tech Support

For help using a TI-83/84 graphing calculator to enter then graph relations and use the Table Feature, see Appendix B-1, B-2, and B-6. If you are using a TI-nspire, see Appendix B-37, B-38, and B-42.



I graphed the equation using these window settings because I knew that the y -intercept would be at 400 and the x -intercept would be at 300.



| X | Y1 |
|--------|--------|
| 171.00 | 172.00 |
| 172.00 | 170.67 |
| 173.00 | 169.33 |
| 174.00 | 168.00 |
| 175.00 | 166.67 |
| 176.00 | 165.33 |
| 177.00 | 164.00 |

X=175

I set the decimal setting to two decimal places because x and y represent money. Then I created a table of values.

- b) Since the **degree** of the equation is one and the graph is a straight line, the relationship is linear. The first differences in the table are constant.

In the table, each increase of 1 in the x -values results in a decrease of about 1.33 in the y -values.

| L1 | L2 | L3 |
|-----|--------|-------|
| 171 | 172 | -1.33 |
| 172 | 170.67 | -1.34 |
| 173 | 169.33 | -1.33 |
| 174 | 168 | -1.33 |
| 175 | 166.67 | -1.34 |
| 176 | 165.33 | -1.33 |
| 177 | 164 | -1.33 |

L3(7) =

- c) By tracing up and down the line, or by scrolling up and down the table, Patrick can see the combinations of pounds and euros. He can use the equation, in either form, to calculate specific numbers of pounds or euros.

Tech Support

For help creating a difference table with a TI-83/84 graphing calculator, see Appendix B-7. If you are using a TI-*n*spire, see Appendix B-43.

EXAMPLE 3

Selecting a representation for a linear relation

Judy is considering two sales positions. Sam's store offers \$1600/month plus 2.5% commission on sales. Carol's store offers \$1000/month plus 5% commission on sales. In the past, Judy has had about \$15 000 in sales each month.

- Represent Sam's offer so that Judy can check what her monthly pay would be.
- Represent the two offers so that Judy can compare them. Which offer pays more?

Justine's Solution

- a) Let x represent her sales in dollars.
Let y represent her earnings in dollars.
An equation will help Judy check her pay.

$$y = 1600 + 0.025x$$

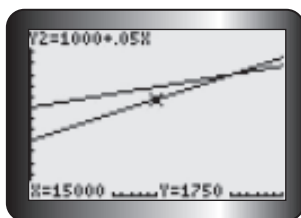
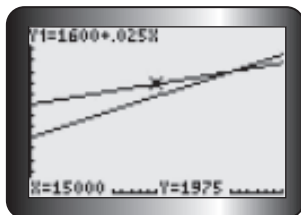
I chose letters for the variables. I wrote an equation to describe what Judy's monthly pay would be. Her base salary is \$1600. Her earnings for her monthly sales would be $0.025x$, since $2.5\% = \frac{2.5}{100}$ or 0.025.



Tech Support

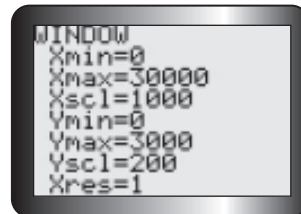
For help changing the window settings and tracing on a graph using a TI-83/84 graphing calculator, see Appendix B-4 and B-2. If you are using a TI-nspire, see Appendix B-40 and B-38.

- b) The equation for Carol's offer is $y = 1000 + 0.05x$.
Judy can use a graph to compare her pay for a typical month.



I wrote an equation for Carol's offer and graphed both relations using a graphing calculator.

I adjusted the settings, as shown, so I could see the point where the graphs crossed.



I used Trace to compare the two offers.

Sam's offer pays more.

In Summary

Key Idea

- Three useful ways to represent a linear relation are
 - a table of values
 - a graph
 - an equation

Need to Know

- A linear relation has the following characteristics:
 - The first differences in a table of values are constant.
 - The graph is a straight line.
 - The equation has a degree of 1.
- The equation of a linear relation can be written in a variety of equivalent forms, such as
 - standard form: $Ax + By + C = 0$
 - slope y-intercept form: $y = mx + b$
- A graph and a table of values display some of the ordered pairs for a relation. You can use the equation of a relation to calculate ordered pairs.

CHECK Your Understanding

1. Which of these ordered pairs are not points on the graph of $2x + 4y = 20$? Justify your decision.
a) (10, 0) b) (-3, 7) c) (6, 2) d) (0, 5) e) (12, -1)

2. Jacob has \$15 to buy muffins and doughnuts at the school bake sale, as a treat for the Camera Club. Muffins are 75¢ each and doughnuts are 25¢ each. How many muffins and doughnuts can he buy?
 - a) Create a table to show the possible combinations of muffins and doughnuts.
 - b) What is the maximum number of muffins that Jacob can buy?
 - c) What is the maximum number of doughnuts that he can buy?
 - d) Write an equation that describes Jacob's options.
 - e) Graph the possible combinations.
3. Refer to question 2. Which representation do you think is more useful for Jacob? Justify your choice.

PRACTISING

4. State two ordered pairs that satisfy each linear relation and one ordered pair that does not.

| | |
|-------------------|--------------------|
| a) $y = 5x - 1$ | c) $y = -25x + 10$ |
| b) $3x - 4y = 24$ | d) $5x = 30 - 2y$ |
5. Define suitable variables for each situation, and write an equation.
 - a) Caroline has a day job and an evening job. She works a total of 40 h/week.
 - b) Caroline earns \$15/h at her day job and \$11/h at her evening job. Last week, she earned \$540.
 - c) Justin earns \$500/week plus 6% commission selling cars.
 - d) Justin is offered a new job that would pay \$800/week plus 4% commission.
 - e) A piggy bank contains \$5.25 in nickels and dimes.
6. Graph the relations in question 5, parts a) and b).
7. Refer to question 5, parts c) and d). Justin usually has about \$18 000 in weekly sales. Should he take the new job? Justify your decision.
8. Deb pays 10¢/min for cell-phone calls and 6¢/text message. She has a budget of \$25/month for both calls and text messages.
 - a) Create a table to show the ways that Deb can spend up to \$25 each month on calls and text messages.
 - b) Create a graph to show the information in the table.
9. Leah earns \$1200/month plus 3.5% commission.
 - a) Create an equation that she can use to check her paycheque each month.
 - b) Last month, Leah had \$96 174 in sales. Her pay before deductions was \$4566.09. Is this amount correct? Explain your answer.

10. Ben's Bikes rents racing bikes for \$25/day and mountain bikes for \$30/day. Yesterday's rental charges were \$3450.
- Determine the greatest number of racing bikes that could have been rented.
 - Determine the greatest number of mountain bikes that could have been rented.
 - Write an equation and draw a graph to show the possible combinations of racing and mountain bikes rented yesterday.



11. Abigail is planning to fly to Paris and then travel through Switzerland and Austria to Italy by train. On the day that she goes to buy the foreign currencies she needs, one euro costs \$1.40 and one Swiss franc costs \$0.90. What combinations of these currencies can Abigail buy for \$630? Use two different strategies to show the possible combinations.
12. A student council invested some of the money from a fundraiser in a savings account that pays 3%/year and the rest of the money in a government bond that pays 4%/year. The investments earned \$150 in the first year.
- Define two variables for the information, and write an equation.
 - Graph the information.
13. Maureen pays a \$350 registration fee and an \$85 monthly fee to belong to a fitness club. Lia's club has a higher registration fee but a lower monthly fee. After five months, both Maureen and Lia have paid \$775. Determine the possible fees at Lia's club.
14. a) Use the chart to show what you know about linear relations.

| | |
|--|----------------------------|
| Characteristics: | Methods of Representation: |
| Examples: | Non-examples: |
| <div style="border: 1px solid red; border-radius: 50%; padding: 5px; display: inline-block;">Linear Relation</div> | |

- List the advantages and disadvantages of each of the three ways to represent a linear relation. Describe situations in which each representation might be preferred.

Extending

15. Create a situation that can be represented by each equation.
- $0.10x + 0.25y = 4.65$
 - $y = 900 + 0.025x$
16. Allan plans to create a new coffee blend using Brazilian beans that cost \$12/kg and Ethiopian beans that cost \$17/kg. He is going to make 150 kg of the blend and sell it for \$14/kg. Write and graph two equations for this situation.