IM2 Problem Set 6.7 - Working with Quadratic Functions

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic functions? • How can I use graphs and equations of quadratic relations to make predictions from data sets & their models

1. (CI) Expand and simplify:
   a. (i) $(x - 11)(x + 11)$ (ii) $-2(x - 6)(x + 6)$ (iii) $(4x - 9)(4x + 9)$
   b. (i) $(x - 8)^2$ (ii) $(9x - 2)^2$ (iii) $(2x + 5)^2$
   c. (i) $(2x - 1)(x + 3)$ (ii) $(4x + 5)(6x + 2)$ (iii) $(3x - 1)(3x + 2)$

2. (CI) Factor the following quadratic expressions:
   a. (i) $x^2 - 4x - 32$ (ii) $x^2 - 10x + 25$ (iii) $x^2 - 6x$
   b. (i) $2x^2 - x - 6$ (ii) $9x^2 - 6x - 1$ (iii) $3x^2 + 10x - 25$
   c. (i) $x^2 - 100$ (ii) $4x^2 - 100$ (iii) $16x^2 - 121$

3. (CA) Mr S throws a ball upwards from the roof a building which has a height of 25m. The ball reaches a maximum height of 45m two seconds later and hits the ground five seconds after being thrown.

   a. Draw an accurate sketch of the relation between the height of the ball and its time in flight.
   b. Where are the zeroes of the function?
   c. What at the coordinates of the vertex?
   d. Determine an equation that models this relationship?
   e. State the domain and range of the relation.

4. (CI) Another two questions about rectangles:

   a. One side of a rectangle is 4 m shorter than three times the other side. Find the sides if the perimeter of the rectangle is 48 m.
   b. One side of a rectangle is 4 m shorter than three times the other side. Find the sides if the area of the rectangle is 319 m^2 .

5. (CI) Given the quadratic function $f(x) = x^2 + 3x - 18$, determine the zeroes, y-intercept and the vertex and then sketch the parabola.

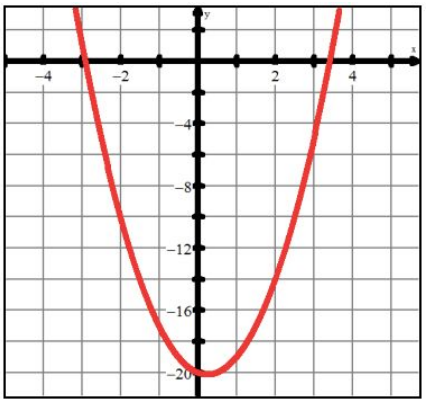
6. (CA) Determine the equations of the parabolas from the information given in any form that is most convenient and then rewrite each equation in standard form.

   a. The function $h(x)$ has $h(-1) = h(11) = 0$ and the minimum value of $h(x)$ is -72.
   b. The function $g(x)$ has zeroes at $x = 4$ and $x = 9$ and $g(0) = -72$
   c. The function $f(x)$ has an optimum point at $f(-2) = 12$ and a zero at $x = -4$.

7. (CI) The profits of a company in its first 13 months of operations are modeled by the quadratic function $P(m) = -\frac{1}{4}m^2 + 3m - 5$ where m is the number of months (and $m = 1$ represents January) and $P(m)$ is the profit measured in billions of Egyptian pounds.

   a. Evaluate $P(2)$ and interpret.
   b. Determine the month in which the company maximizes its profits and what was the maximum profit?
   c. Determine when the company breaks even and hence solve $P(m) < 0$.
   d. State the domain and range of this profit function. Explain your reasoning.
   e. Solve $P(m) = -12$ and explain why the profits might be negative.

8. (CA) Given the quadratic function $f(x) = 2x^2 - x - 20$ (graphed here), use the TI-84 to help you find necessary information and hence rewrite $f(x)$ in both factored form and vertex form.



9. (CI) Find the intersection point(s) of the following functions. Then graph the two functions on your calculator to verify your answer(s).

   a. $f(x) = 5x + 7$ and $g(x) = x^2 + 2x + 3$
   b. $f(x) = -2x + 5$ and $g(x) = 10 + 2x - x^2$

EXTENSION PROBLEMS

10. The equation $ax^2 + 5x = 3$ has $x = 1$ as a solution. What is the other solution?

11. Find all x such that $\frac{10}{x^2} + \frac{22}{x} + 4 = 0$.

12. Find all solutions to $2w^4 - 5w^2 + 2 = 0$.

13. Find all solutions to the equation $\frac{x-6}{x-5} = \frac{4}{x-2}$.