IM2 Problem Set 6.4 - Working with Quadratic Functions

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic functions? • How can I use graphs and equations of quadratic relations to make predictions from data sets & their models

1. (CA) Use your calculator to graph the following parabolas and then use the calculator to find the (i) vertex and (ii) the zeroes. Finally, rewrite each equation in ***factored form*** and in ***vertex form***.

   a. $f(x) = x^2 - 2x - 8$ b. $g(x) = 2x^2 - 4x$ c. $h(x) = 3x^2 + 9x - 30$

2. (CI) Given the pattern …..24, 21, 16, 9, 0, -11, -24, ….

   a. How do you know the pattern is NOT linear?
   b. How do you know the pattern is NOT exponential?
   c. What are the next three terms of the sequence
   d. What are the 3 terms that came **before** 24?

3. (CI) Using this pattern from Q2 (…..24, 21, 16, 9, 0, -11, -24, ….), we create the following data set:

x	4	5	6	7	8	9	10
$f(x)$	24	21	16	9	0	-11	-24

   a. At which x coordinate will the second x-intercept be?
   b. Find the vertex of the parabola.
   c. Using the two forms of the quadratic equations, write equations for this quadratic function.
   d. Evaluate the following: $f(11)$, $f(12)$, $f(13)$, and $f(14)$
   e. Hence, evaluate the following:
      i. (i) $[f(14) - f(13)] - [f(13) - f(12)]$
      ii. (ii) $[f(13) - f(12)] - [f(12) - f(11)]$
      iii. Explain your observations in these calculations

4. (CI) Use distribution to simplify the following products:

   a. (i) $(r + 1)(r - 3)$ (ii) $(k - 2)(k - 3)$ (iii) $(g - 5)^2$ (iv) $(2x - 1)^2$
   b. (i) $(3p - 3)(p - 1)$ (ii) $(2x - 3)(3x + 3)$ (iii) $(4n + 4)(5n - 8)$

5. (CI) Two parabolas have zeros of 1 and 11. One has a maximum value of 12 and the other has a minimum value of -6. Sketch the two parabolas on the same axes and then determine their equations.

6. (CA) A penguin dives into a lake to catch a fish. The underwater path of the penguin is described by the model $d(x) = \frac{1}{2}x^2 - 3x$, where x represents the horizontal position of the penguin relative to its entry point and d is the depth of the penguin underwater. Both measurements are in meters.
    a. Graph the parabola on your calculator. State your window settings.
    b. Explain what the point (2, -4) represents in the context of this problem.
    c. State the domain and range in the context of this problem.
    d. What is the greatest depth below the water surface?
    e. Factor the equation $y = \frac{1}{2}x^2 - 3x$.

7. (CI) The vertex of a parabola is at (4,-2) and the parabola goes through the point (0,6). Determine:
    a. if the relation has a maximum or minimum value?
    b. the equation of the quadratic relation.
    c. the coordinates of the x-intercepts.
    d. Sketch the parabola.

8. (CI) The following quadratic functions are written in standard form ($y = ax^2 + bx + c$). Rewrite the following equations in factored form. The process you are carrying out is called ***factoring***.
    a. (i) $y = x^2 - x - 6$ (ii) $y = x^2 + x - 12$ (iii) $y = x^2 + 5x + 4$ (iv) $y = x^2 - 4x - 32$
    b. (i) $y = 2x^2 - 2x - 12$ (ii) $y = 3x^2 + 24x + 45$

9. (CA) A company models the profit of its latest video game using the relation $P(x) = -4x^2 + 20x - 9$, where x is the number of games produced (in hundreds of thousands) and P is the profit in millions of dollars.
    a. Explain what the point (5,-9) means in the context of this problem.
    b. Suggest a reasonable domain for this relation, given the context of the problem.
    c. What are the break even points for the company?
    d. What is the maximum profit that the company can earn?
    e. How many games must be produced to earn this maximum profit?
    f. Rewrite the equation in vertex form.

EXTENSION PROBLEMS

10. Write the product of two binomials such that the product is equal to zero when $x = 3$ and $x = -5$.
11. 100 times my number is equal to the square of my number divided by 4. What are the possible values of my number?
12. The sum of the roots of a quadratic equation is equal to -3, while the product of the roots is -40. Find the equation of this quadratic as well as finding the values of the two roots.