IM2 Problem Set 6.3 - Working with Quadratic Functions

BIG PICTURE of this UNIT:	● How do we analyze and then work with a data set that shows both increase and decrease ● What is a parabola and what key features do they have that makes them useful in modeling applications ● How do I use graphs, data tables and algebra to analyze quadratic functions? ● How can I use graphs and equations of quadratic relations to make predictions from data sets & their models

1. (CA) For the following quadratic equations,
   a. state the location of the vertex and the "direction of opening"
   b. expand and simplify to express the equation in standard form.
   (i) $y = (x - 6)^2 + 5$ (ii) $y = 5 + 2(x + 3)^2$ (iii) $y = \frac{1}{2}(x - 5)^2$

2. (CI) The following quadratic equations have been given to you in standard form (as I already have expanded and simplified them.) Now work **backwards** to find out what expressions I expanded in order to get these equations i.e. the standard equation of $y = x^2 + 2x - 8$ came from the expansion of $y = (x - 2)(x + 4)$.
   (i) $y = x^2 - x - 6$ (ii) $y = x^2 + 6x - 16$ (iii) $y = x^2 + 4x + 3$

3. (CI) Apply the distributive property to simplify the following polynomial expressions:

   a. $2(2x + 3)(x - 4)$ b. $-3(x + 5)(2x - 1)$ c. $-(2x + 3)(5 - x)$ d. $-\frac{1}{2}(x + 6)(2x - 1)$

4. (CA) Use a graphing calculator (or use www.desmos.com) to graph $y = \boldsymbol{a}(x - 2)(x + 6)$ when $\boldsymbol{a} = 3$.
   a. Describe what happens to the graph as you change the value of $\boldsymbol{a}$ to 2, 1, ½, ¼ , 0, -1, -2, -3. Include sketches.
   b. Where is the axis of symmetry in each parabola?
   c. Finally, what does $\boldsymbol{a}$ "control" in the graph of a parabola?

5. (CA) A baseball is thrown from the top of building and it falls to the ground below. Its height above the ground depends upon the time elapsed according to the model $h(t) = -5t^2 + 5t + 30$, where h is height in meters and t is the elapsed time in seconds.
   a. Graph the function and state your window settings.
   b. How tall is the building?
   c. When does the ball hit the ground?
   d. When does the ball reach its maximum height? What is it maximum height?
   e. Rewrite the model using factored form and using the vertex form of a quadratic function.

6. (CI) The zeroes of a parabola are -3 and 5. The graph crosses the y-axis at -75. Determine:

   a. if the relation has a maximum or minimum value?
   b. the equation of the quadratic relation.
   c. the coordinates of the vertex.
   d. Sketch the parabola.

7. (CI) Common Factors. Identify the greatest factor that is common to all terms in the following algebraic expressions and then factor the expression using the identified GCF.

   a. (i) $2L + 2W$ (ii) $30x + 24y$ (iii) $20x - 10$
   b. (i) $9x^2 + 3x + 15x^3$ (ii) $24mn - 16m^2n$ (iii) $3x^2 + 6x - 18$ (iv) $4(x - 3) + x(x - 3)$

8. (CI) A ball is tossed straight up into the air. Its height is recorded every quarter of second.

Time (s)	0	0.25	0.5	0.75	1.00	1.25	1.50	1.75	2.00
Height (m)	1.5	3.5	4.9	5.7	5.7	5.2	4.1	2.4	0.1

   a. Draw the scatter-plot on graph paper.
   b. Draw the graph that best fits the data.
   c. Using your graph, what is the maximum height of the ball and when does it reach that maximum height?
   d. Using your graph, when does the ball reach the ground?
   e. Using your graph and your answers from the previous questions, determine an equation for this relationship.
   f. (CA) Use your calculator to determine the regression equation for the relationship.

9. (CA) The axis of symmetry of a parabola can be determined using the "formula" $x = -\frac{b}{2a}$. For the following quadratic functions, (i) use the formula to find the axis of symmetry and then (ii) graph them on the calculator to find the vertex.
   a. (i) $y = x^2 + 6x - 8$ (ii) $y = x^2 - 12x + 20$ (iii) $y = x^2 + 7x - 1$
   b. (i) $y = 2x^2 + 6x - 3$ (ii) $y = 2x^2 - 12x + 3$ (iii) $y = 2x^2 - 9x - 11$

EXTENSION PROBLEMS

10. https://undergroundmathematics.org/quadratics/which-parabola/download/problem.pdf

11. Create 2 rectangles so that the first one has a bigger perimeter and the second has a bigger area.

12. Create 2 rectangles so that the first rectangle has exactly twice the perimeter of the second rectangle AND the second rectangle has exactly twice the area of the first rectangle.