IM2 Problem Set 6.2 - Working with Quadratic Functions

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic functions? • How can I use graphs and equations of quadratic relations to make predictions from data sets & their models

1. (CI) Simplify (using the distributive property) the following polynomial expressions:
    a. $3(2x^2 - 1) + 6(2x - 3) - (2x^2 - 5x)$
    b. $6(x + 5) - 2(x^2 + 4) + x(x - 5)$
    c. $3(2y^2 - 1) + 6(2y - 3) - (2y^2 - 5y)$

2. (CA) For the quadratic function $g(x) = 2(x + 3)(x - 4)$, determine:
    a. The direction of opening of the parabola.
    b. The zeroes (also known as the x-intercepts or roots of the equation)
    c. The axis of symmetry
    d. The optimal point (also known as the vertex)
    e. The y-intercept
    f. Include a sketch of the parabola.
    g. Explain how you could graph this parabola WITHOUT a calculator.

3. (CA) Numeracy Skills: Find the two numbers that:
    a. will add to a sum of -1 and multiply to product of -6.
    b. will add to a sum of -2 and multiply to product of -8.
    c. will add to a sum of 11 and multiply to product of 18.
    d. will add to a sum of -11 and multiply to product of 24.
    e. will add to a sum of -7 and multiply to product of -30.
    f. will add to a sum of 10 and multiply to product of 25.
    g. will add to a sum of 8 and multiply to product of 7.
    h. will add to a sum of -19 and multiply to product of 48.
    i. will add to a sum of 22 and multiply to product of 121.

4. Given the pattern …..16,15,12,7,0,-9,-20 …..
    a. How do you know the pattern is NOT linear?
    b. How do you know the pattern is NOT exponential?
    c. What are the next three terms of the sequence?
    d. What are the 3 terms that came **before** 16?

5. (CI) Apply the distributive property to simplify the following polynomial expressions:
   a. $(x + 3)(2x + 4)$ b. $(y + 2)(y - 1)$ c. $(2x + 3)(3x - 5)$ d. $(x - 4)(2x - 5)$

6. The SAMSOONG company introduces a new cellphone and its PROFITS are modelled by the equation $P(m) = -5m^2 + 80m - 100$ where m is time in months and $P(m)$ is the profit in millions of dollars. The cellphone is sold for a period of 2 years.
   a. Graph the profit function on your TI-84 and state your window settings.
   b. Calculate the zeroes of the quadratic and interpret what they mean.
   c. Write the equation in factored form $y = a(x - R)(x - S)$, given your answer in (b).
   d. Calculate the coordinates of the vertex and interpret.
   e. Write the equation in vertex form $y = a(x - h)^2 + k$, given your answer in (d).
   f. Evaluate $P(5)$ and interpret.
   g. Solve $P(m) = -25$ and interpret
   h. Solve $P(m) < 0$ and interpret
   i. For what values of m are the profits DECREASING?

7. (CI) Find value(s) for x that make the following equations true:
   a. $0 = 2x + 6$
   b. $0 = (2x + 6)(x - 3)$
   c. $0 = x^2 - 3x - 4$
   d. $0 = 2x^2 - x - 6$
   e. $0 = (x + 3)^2 - 9$

8. (CA) For the following quadratic functions, (i) $f(x) = 2(x + 2)^2 - 18$ and $g(x) = -2(x - 5)^2 + 8$.
   a. Graph them on your calculator.
   b. Find the vertex.
   c. Explain why we call this form of a quadratic equation "vertex form."
   d. Find the x-intercepts.
   e. How would you find the x-intercepts if you did NOT have access to a graphing calculator?

9. (CA) You are provided with data showing relationship between how fast a car travels (in miles per hour) and its fuel consumption (miles per gallon).

speed	15	20	25	30	35	40	45	50	55	60	65	70	75
mileage	22.3	25.5	27.5	29.0	28.8	30.0	29.9	30.2	30.4	28.8	27.4	25.3	23.3

   a. Explain what the point (30,29.0) means in the context of the problem.
   b. Graph the scatter plot on your calculator. Record your window settings.
   c. Find a quadratic regression equation for the data.
   d. What speed will optimize the mileage?
   e. How fast should I travel if I wish to have a fuel consumption of 24 miles per gallon?

EXTENSION PROBLEMS

10. (CI) You are given a rectangle, where the length is given by the expression $x + 5$ and the width is given by the expression $x - 2$.

   a. Determine the area of the rectangle if $x = 15$ cm
   b. Determine the area of the rectangle if $x = 22$ cm.
   c. Can you determine the area of the rectangle if $x = 1$ cm?
   d. Find an expression for the area of the rectangle as a function of x (i.e. $A(x) = $???)
   e. What would be a possible domain and range for this area function?
   f. A second rectangle has an area function given as $A(x) = x^2 - 2x - 35$. Determine expressions for the length and the width of this rectangle.

11. (CA) A company's profit, in thousands of dollars, on sales of computers is modelled by the function $P(x) = -2(x - 3)^2 + 50$, where x is in thousands of computers sold. The company's profits, in thousands of dollars, on sales of phones is modelled by the function $P(x) = -(x - 2)(x - 7)$, where x is in thousands of phones sold. Calculate the maximum profit the business can earn and describe how it can earn this maximum profit.

12. https://nrich.maths.org/11011