IM2 Problem Set 5.7 - Working with Exponential Functions

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CA)** Generating Data Sets. Heads or Tails Activity ⇒ Modeling Exponential Growth H&T Activity. The purpose of this activity is to provide a simple model to illustrate exponential growth of cancerous cells. In our experiment, a HEAD on a COIN TOSS represents a cancerous cell. If the COIN lands HEADS side up, the cell divides into the "parent" cell and "daughter" cell. The cancerous cells divide like this uncontrollably-without end. We will conduct 10 trials and record the number of "cancerous cells".

   Exponential Growth Procedure

   a. Use either the website http://www.shodor.org/interactivate/activities/Coin/ OR https://www.random.org/coins/ to toss our coins. We will start with 2 coins. This is trial # 0.
   b. Count the number of HEADS that appear (recall these are cancerous cells) For every coin with the HEAD side showing, add another coin and then record the new population. (Ex. If 5 coins land HEADS, then you add 5 more coins)
   c. Repeat step number 2 until you are done with 10 trials.
   d. Add your results to this class data table. Use the GROWTH ACTIVITY spreadsheet.

      https://docs.google.com/spreadsheets/d/18l6kKZd0CcbYl24-o7Quxcwf_MRVSXZdWj3qb1Vn1GU/edit?usp=sharing

Table of Results

Trial #	0	1	2	3	4	5	6	7	8	9	10
# of coins	2										

Prediction #1 ⇒ What would you predict for the # of coins for trials 11 and 12? Make your prediction and then test it out.

Prediction #2 ⇒ If our "cancer" becomes detectable when there are 10,000 cells, how many trials of our experiment would this take?

2. **(CI)** Radioactivity Simulation ⇒ Modeling Exponential Decay Activity. The purpose of this activity is to provide a simple model to illustrate exponential decay of radioactive material. In our experiment, let's say that Mrs Knox accidentally spilt some radioactive molecules in her lab, so our building is now UNSAFE and we must evacuate. So, to simulate the decay of a radioactive material, a DICE ROLL of 6 represents a DECAY activity i.e a molecule "changes" form èfrom an "unsafe radioactive form" to a "safe non-radioactive form". If the DICE lands showing a 6, the molecule decays into a non-radioactive form. We will conduct up to 15 trials and record the number of remaining "unsafe radioactive molecules".

   Exponential Decay Procedure

   a. Use the website http://www.roll-dice-online.com/ to roll our dice. We will start with 99 dice. This is trial # 0.
   b. Count the number of 6s that appear (recall these are safe non-radioactive molecules) For every dice showing a 6, remove that dice and then record the new population. (Ex. If 5 dice showing 6s, then you remove 5 dice)
   c. Repeat step number 2 until you are done with 10 trials.
   d. Add your results to a class data table. Use the GROWTH DECAY spreadsheet.

      https://docs.google.com/spreadsheets/d/18l6kKZd0CcbYl24-o7Quxcwf_MRVSXZdWj3qb1Vn1GU/edit?usp=sharing

Table of Results

Trial #	0	1	2	3	4	5	6	7	8	9	10
# of dice	99										

Prediction #1 ⇒ What would you predict for the # of dice for trials 11 and 12? Make your prediction and then test it out.

Prediction #2 ⇒ Let's say that our scenario becomes safe OVERALL when there are only 2 unsafe, radioactive molecules left, how many trials of our experiment would this take?

3. **(CI)** An Exponential equation has the form $\boldsymbol{y = a(b)^x}$ or $\boldsymbol{y = a(1 + r)^x}$, where $\boldsymbol{a}$ = initial value, $\boldsymbol{b}$ is the growth factor/common ratio. (It turns out that $\boldsymbol{b} = \mathbf{1} + \boldsymbol{r}$, where $\boldsymbol{r}$ is the decimal value of % increase given). For the following equations, (i) decide if they can be used to model growth or decay and (ii) determine the rate at which the change happens.

   a. (i) $y = 400(1.75)^x$ (ii) $y = 100(0.75)^x$ (iii) $y = 100(0.995)^x$
   b. (i) $y = 1{,}000(0.30)^x$ (ii) $y = 2500(1.5)^x$ (iii) $y = 50(1 + \frac{0.25}{6})^x$

4. **(CA)** Examining Changes in the Compounding Conditions. When my oldest son, Alexander, was born, I invested \$5,000 in an education fund for him. The education fund is earning 8% compound interest every year. You will develop an answer to my questions:

   a. How much this investment is worth when Alexander starts university at the age of 19?
   b. When has the investment tripled its value?

   When interest is "paid" to the investor, it DOES NOT HAVE TO BE ANNUALLY!!!. What if an investor (like me) wants the interest paid MORE FREQUENTLY? How does this change the value of an investment?? How does it change the formula that I can use to predict future values?
   Let's reconsider my first example: When my oldest son, Alexander, was born, my wife and I invested \$5,000 in an education fund for him. The education fund is earning 8% interest every year
   ⇒ Now I will have 4 investment options that you will investigate:

OPTION A ⇒ 8%/a compounded semi-annually
OPTION B ⇒ 8%/a compounded quarterly
OPTION C ⇒ 8%/a compounded monthly
OPTION D ⇒ 8%/a compounded daily

   c. Summary

      i. Does the value of my investment for Alex change in value given the different compounding conditions? Any ideas as to WHY/WHY NOT?

      ii. Does the time taken to triple my investment change given the different compounding conditions? Any ideas as to WHY/WHY NOT?

      iii. Does the formula I use to predict future values change given the different compounding conditions?

5. **(CA)** For each situation, determine: (i) the amount (value of the investment) (ii) the interest earned

   a. \$4000 borrowed for 4 years at 3%/a, compounded annually
   b. \$7500 invested for 6 years at 6%/a, compounded monthly
   c. \$15 000 borrowed for 5 years at 2.4%/a, compounded quarterly
   d. \$28 200 invested for 10 years at 5.5%/a, compounded semi-annually
   e. \$850 financed for 1 year at 3.65%/a, compounded daily
   f. \$2225 invested for 47 weeks at 5.2%/a, compounded weekly