IM2 Problem Set 5.6 - Working with Exponential Functions

BIG PICTURE of this UNIT:	- How can we analyze growth or decay patterns in data sets & contextual problems? - How can we algebraically & graphically summarize growth or decay patterns? - How can we compare & contrast linear and exponential models for growth and decay problems. - How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CA)** The half-life of a medication is the amount of time for half of the drug to be eliminated from the body. The half-life of *Advil* or ibuprofen is represented by the equation $R(t) = M\left(\frac{1}{2}\right)^{\frac{t}{2}}$, where R is the amount of Advil remaining in the body, M is the initial dosage, and t is time in hours since a dose was taken.

   a. A 200 milligram dosage of Advil is taken at 11:00 am. How many milligrams of the medication will remain in the body at 5:00 pm?
   b. Mr R is taking an Advil every 12 hours and he takes a 200 milligram dosage of Advil at 11:00 am, how many milligrams of the medication will remain in the body 12 hours later.
   c. He then takes another dose at 11:00 pm, how many milligrams of the medication will be in his body at that time?

2. **(CI)** Evaluate the following expressions:

   a. (i) $8^{-\frac{2}{3}}$ (ii) $25^{-\frac{3}{2}}$ (iii) $16^{-\frac{5}{4}}$ (iv) $81^{-\frac{3}{4}}$
   b. (i) $3^{\frac{4}{3}} \times 3^{\frac{5}{3}}$ (ii) $\left(7^{3}\right)^{\frac{2}{3}}$ (iii) $8^{-\frac{5}{3}} \times 8^{\frac{6}{3}}$

3. **(CI)** Given the function $g(x) = 16 - 2^{x+2}$:

   a. Evaluate $g(-3)$, $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, $g(2)$
   b. determine the x- and y-intercept(s) - if they exist
   c. determine the equation of the asymptote of $g(x)$
   d. sketch $g(x)$, labelling the data points and intercept(s) and the asymptote.

4. **(CA)** A tool & die business purchased a piece of equipment of $250,000. The value of the equipment depreciates at a rate of 12% each year.

   a. Write an exponential decay model for the value of equipment.
   b. What is the value of equipment after 5 years?
   c. Estimate when the equipment will have a value of $70,000
   d. What is the monthly rate of depreciation

5. **(CI)** The expression $7^{\frac{1}{3}}$ can be rewritten in radical form as $\sqrt[3]{7}$ and the expression $7^{\frac{2}{3}}$ is rewritten as either $(\sqrt[3]{7})^2$ or $\sqrt[3]{7^2}$. Rewrite each exponential expression in radical form (and vice versa in Qb).

   a. (i) $5^{\frac{1}{2}}$ (ii) $4^{\frac{4}{3}}$ (iii) $2^{\frac{5}{3}}$ (iv) $7^{\frac{4}{3}}$
   b. (i) $(\sqrt{10})^3$ (ii) $\sqrt[6]{2}$ (iii) $\sqrt[4]{2^5}$ (iv) $(\sqrt[4]{6})^5$

6. **(CA)** Percent Change Analysis of a Data Set. Mr S. gives you this data set and is asking you to analyze patterns in the data set in order to determine an equation in the form of $f(x) = ab^x$.

x	-2	-1	0	1	2	3	4
$f(x)$	28 $\frac{4}{9}$	21 ⅓	16	12	9	6.75	5.0625

   a. Determine the "percent change" between each pair of terms:
   % change = $\frac{y_2 - y_1}{y_1}$; % change = $\frac{y_3 - y_2}{y_2}$; % change = $\frac{y_4 - y_3}{y_3}$; etc ….
   b. This creates an equation in the form of $y = a(1 + r)^x$. Use a data point to find the value of a and now, what equation models this data set?
   c. Secondly, now determine the "common ratio" between each pair of terms (you do this by dividing the successive y terms ==> ratio = $\frac{y_2}{y_1}$; ratio = $\frac{y_3}{y_2}$; r = $\frac{y_4}{y_3}$; etc …..
   d. Finally, what is the equation for this data set?

Part 2 - Skills/Concepts Application Problems

7. **(CA)** From 1990 to 1997, the number of cell phone subscribers S (in thousands) in the US can be modeled by the equation $S = 5535.33(1.413)^t$ where t is number of years since 1990.

   a. Identify the growth factor and annual percent increase.
   b. In order to see this function on your TI-84, you need to set appropriate window settings. Record your window settings that you used to see the graph of this model.
   c. In what year was the number of cell phone subscribers about 31 million?
   d. According to the model, in what year will the number of cell phone subscribers exceed 90 million?
   e. Estimate the number of subscribers in 2020
   f. Do you think this model can be used to predict future number of cell phone subscribers? Explain

8. **(CI)** Simplify the following expressions using the appropriate exponent laws and operations.

   a. $\frac{(6x^3y^{-4})^{-2}}{(3x^2y^5)^{-3}}$ b. $\frac{(8x^3y^{-4})^{-2}}{(-4x^{-1}y^2)^{-3}\cdot(2x^5y^{-3})^{-2}}$ c. $\frac{x^{-1}+y^{-1}}{(xy)^{-2}}$

9. **(CA)** In 1990 the cost of attending University of Math was $15000. During the next 25 years, the cost has increased by an average of 5.2% per year.

   a. Write a model giving the cost, $C(t)$, at University of Math t years after 1990.
   b. In what year did the tuition exceed $30,000?
   c. Estimate the tuition in 2020 - the year you will attend this college!
   d. Mr S has set up a college fund for his son Ian. This fund started in 2000 with an initial investment of $30,000 and has grown at 5.8% every year. If Ian attends a three year program at U of Math, can this college fund pay for these costs? Show your work/reasoning.

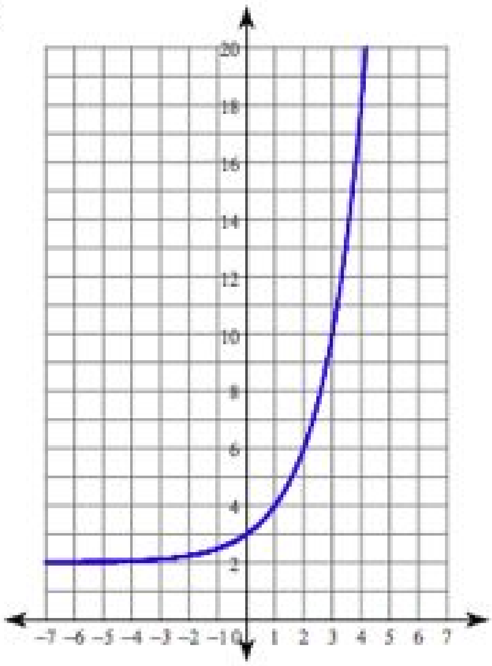
10. **(CI)** Solve the following equations and verify your solutions.

	(i)	(ii)	(iii)
a.	$2^{3-x} = 2^4$	$2^{x-3} = 2^{3x+1}$	$2^{2x+3} = 16$
b.	$2^{1-2x} = 8$	$3^{x+2} = \frac{1}{9}$	$8^x = 16^{x-1}$

11. **(CA)** Mr. S would like to know the equation of the following exponential functions that have been graphed for you.

Write an equation for each graph.

7)



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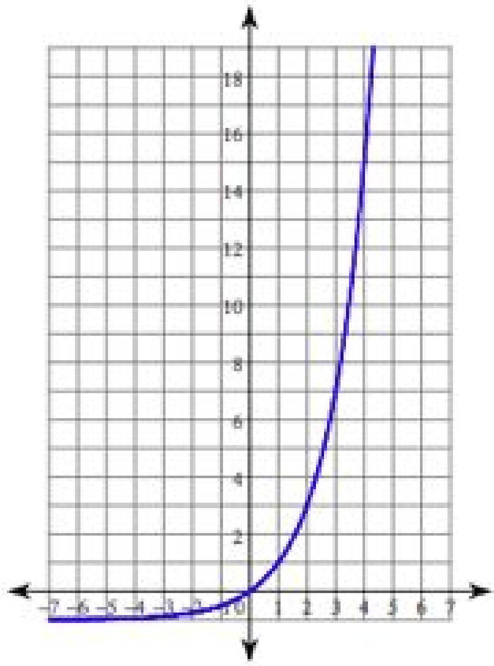

HOMEWORK PROBLEMS:

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2.
3.