IM2 Problem Set 5.5 - Working with Exponential Functions

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CA - DESMOS)** **Investigation #1**

    a. Use DESMOS to graph y = 2^x.
    b. Then graph $y = b^x$ and add a slider for b. Set the slider for b for $1 \le b \le 10$
    c. Play the slider and record observations and describe the effect of "b" on the exponential function
    d. What is changing about the exponential function ⇒ its shape or its location?

2. **(CA - DESMOS)** **Investigation #2**

    a. Use DESMOS to graph y = 2^x.
    b. Then graph $y = b^x$ and add slider. Set the slider for b for $0 \le b \le 1$
    c. Play the slider and record observations and describe the effect of "b" on the exponential function. How is this different that Investigation #1?
    d. What is changing about the exponential function ⇒ its shape or its location?

3. **(CI)** Simplify the following expressions by writing them first as a single power.

    a. $8^{\frac{2}{3}} \times \left(8^{\frac{1}{3}}\right)$ b. $8^{\frac{2}{3}} \div 8^{\frac{1}{3}}$ c. $\left(7^{\frac{5}{6}}\right)^{-\frac{6}{5}}$ d. $\dfrac{9^{-\frac{1}{3}}}{9^{\frac{2}{3}}}$ e. $10^{-\frac{4}{5}} \times 10^{\frac{1}{15}} \div 10^{\frac{2}{3}}$

4. **(CI)** For the following functions, decide if they model growth or decay and then determine the rate at which the change happens.

    a. (i) $y = 400(1.75)^x$ (ii) $y = 100(0.75)^x$ (iii) $y = 100(0.995)^x$ (iv) $y = 1000(0.3)^{-x}$

5. **(CI)** To evaluate rational exponents, use exponent laws to "simplify" first.

    a. To evaluate $8^{\frac{2}{3}}$, Mr D rewrites the expression using exponent laws as $\left(8^{\frac{1}{3}}\right)^2$. Explain how/why this works. What is the value of $8^{\frac{2}{3}}$
    b. Evaluate: (i) $27^{\frac{2}{3}}$ (ii) $100^{\frac{3}{2}}$ (iii) $4^{\frac{5}{2}}$ (iv) $9^{-\frac{3}{2}}$

6. **(CA)** A chemical decays according to the formula $A(t) = 12000\left(\frac{1}{2}\right)^{\frac{t}{25}}$, where t is time in hour and $A(t)$ is the amount of chemical left, measured in grams.

   a. How much chemical is present at the beginning of the experiment?
   b. Evaluate and interpret $A(50)$.
   c. How much of the chemical is left after 10 days?
   d. The idea of a **half-life** is the amount of time it takes for the chemical to decay to half its original amount. Determine the half-life of this chemical.

7. **(CA)** A block of dry ice is losing its mass at a rate of 12.5% per hour. At 1 pm, it weighed 50 kilograms.

   a. What was its weight at 5pm?
   b. What is the approximate half life of dry ice, given the conditions presented in this questions.

Part 2 - Skills/Concepts Application Problems

8. **(CA)** Data Analysis. Use the Ratio Analysis method from PS5.3 and PS5.4 to analyze the following data set.

Year	1950	1960	1970	1980	1990	1995	2000	2005	2010
Population (billions)	2.56	3.04	3.71	4.45	5.29	5.78	6.09	6.47	6.90

   a. What is your prediction for a common ratio?
   b. What equation can you write to model the population?
   c. Use your model to predict the population in 2020.
   d. When will the population be predicted to be double the current amount?

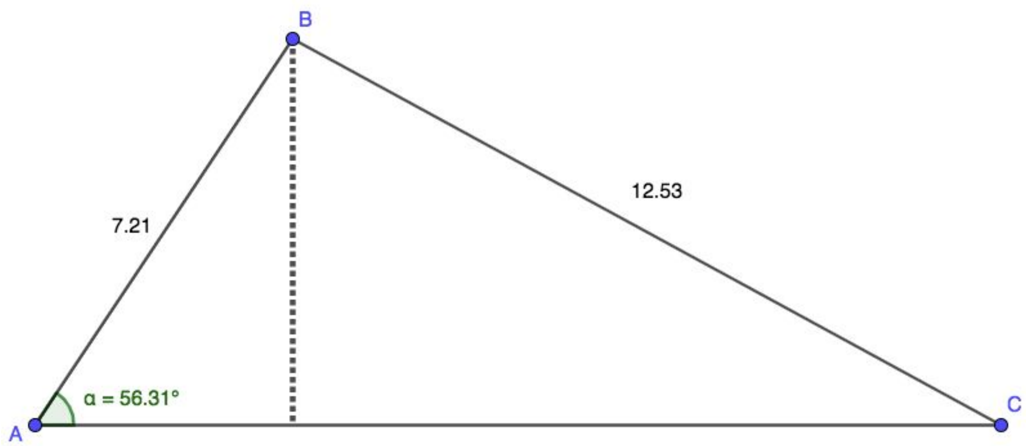
9. **(CA)** Two gemstones were purchased twenty years ago. The emerald stone was purchased for \$500 and the value of this gemstone has been increasing in value each year at a rate of 8% of its previous years value. The garnet stone was purchased for \$1000 and has been increasing in value by \$50 every year.

   a. Determine the value of each stone at $t = 0$, $t = 5$, $t = 10$ and $t = 15$ years from its purchase date.
   b. Determine the value of each stone today.
   c. When will the value of the emerald be twice the value of the garnet?

10. **<u>(CA)</u>** The revenues of a business are modelled by the function $R(t) = 120000(0.95)^t$ and the expenses of the same business are modelled by the function $E(t) = 50000 + 5000t$. In both models, t is time in years since the year the business opened, on January 1st, 2015.

   a. Determine the revenue and expenses of the business in the first 3 years of operation.
   b. Explain how a business determines its profits.
   c. Hence determine the profits that this business makes in each of the first 3 years of operation.
   d. The business will continue to operate until its profits are 0. When will be business operations stop?
   e. How much total profit has the business made at this point?

11. **<u>(CA)</u>** Find the measure of $\angle$ACB as well as the length of CA



<u>HOMEWORK PROBLEMS:</u>

1.
2.
3.