IM2 Problem Set 5.4 - Working with Exponential Functions

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CA)** On January 1st, 2000, Mr S made a deposit of $15,000 in an account pays interest on the balance annually and the account balance is modeled by the function $B(t) = 15000(1.0725)^t$, where t is time in years since January 1st, 2000.

   a. From the equation, how do you know that the model represents a growth curve?
   b. At what rate is the deposit growing?
   c. What was the value of the deposit on Jan 1st, 2019?
   d. What is the value of the deposit now?
   e. In what year will the value of the investment exceed $80,000?

2. **(CA)** Ratio Analysis of a Data Set. Mr S. gives you this data set and is asking you to analyze patterns in the data set in order to determine an equation in the form of $f(x) = ab^x$ for the data set.

x	-2	-1	0	1	2	3	4
$f(x)$	177 $\frac{7}{9}$	133 ⅓	100	75	56.25	42.1875	31.64

   a. Determine the "common ratio" between each pair of terms (you do this by dividing the successive y terms ==> ratio = $\frac{y_2}{y_1}$; ratio = $\frac{y_3}{y_2}$; r = $\frac{y_4}{y_3}$; etc …..
   b. This value for the common ratio is the ***base*** or b in the equation. How can you use the data set to find the value for a?
   c. Finally, what is the equation for this data set?

3. **(CI)** Which of the functions listed below are "growth" functions and which are "decay" functions? For each function, determine the rate at which the growth/decay happens.

   a. $y = 5(2)^x$ b. $y = 100(0.5)^x$ c. $y = 80(1.3)^x$ d. $y = 20(0.8)^x$

4. **(CI)** For each of the functions, state their domain and range. Sketch each function.

   a. (i) $f(x) = 3^x$ (ii) $f(x) = -(3^x)$ (iii) $f(x) = 3^{-x}$ (iv) $f(x) = \left(\frac{1}{3}\right)^x$
   b. (i) $g(x) = 2^x - 3$ (ii) $g(x) = 2^{x-3}$ (iii) $g(x) = 3 - 2^x$ (iv) $g(x) = 2^{-x} + 4$

5. **(CA)** Mr S has been given a new job contract. He will earn $50,000 in the first year of this contract and get a raise of 6% of his previous years' salary (i.e his salary grows by 6% per year)

    a. Write an equation for Mr. S's salary.
    b. Graph the function on your TI-84
    c. What does the *y*-intercept represent?
    d. What would my salary be in 8 years?
    e. After how many years would my salary be $80,000?
    f. What assumption are you making as you answer Qe,f?

6. **(CI)** Mr R has purchased a new car. It cost $50,000 but its value is depreciating at a rate of 12%.

    a. Write an equation for the value of Mr. R's car.
    b. Graph the function on your TI-84.
    c. What does the y-intercept represent?
    d. What would be the value of his car be in 8 years?
    e. After how many years would the value of his car be $10,000?

Part 2 - Skills/Concepts Application Problems

7. **(CA)** Use GEOGEBRA to graph the function $g(x) = 2^x$.

    a. Create a "point on object" ⇒ in other words, put a point onto $g(x)$ using the "point on object" tool
    b. Create a vector.
    c. Now use the "translate by vector" tool and apply it to the function $g(x)$ as well as the point. Describe what happens to both the point and the exponential function.
    d. Write down the new "equation" of the function and explain how the equation of the exponential function is related to the translation vector.

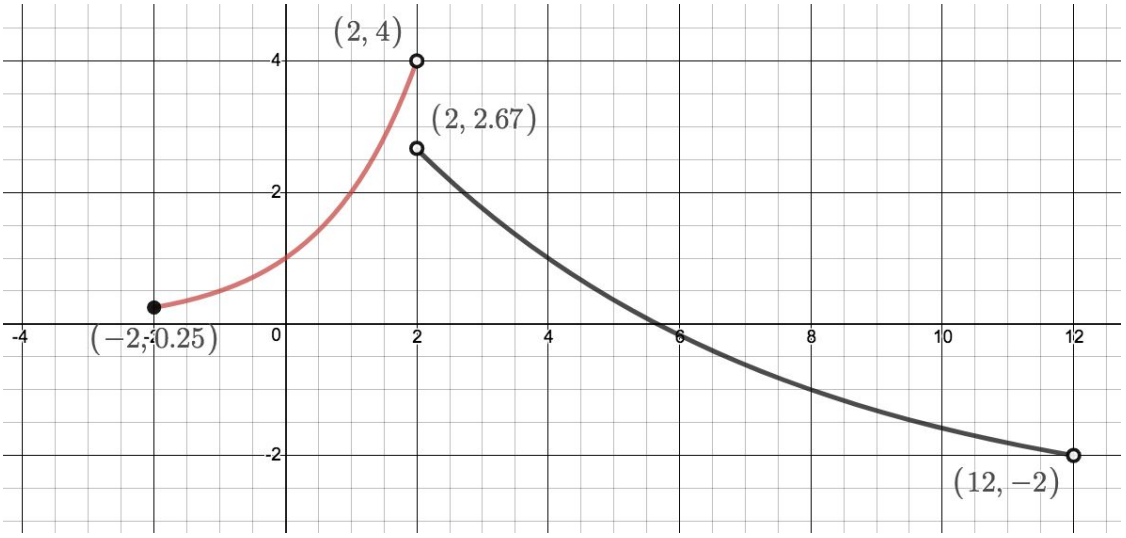
8. **(CI)** Mr D makes the following observation about exponents and exponent rules:

    (i) $8^1 \times 8^1 \times 8^1 = 8^3$ (ii) $8^2 \times 8^2 \times 8^2 = 8^6$ (iii) $8^3 \times 8^3 \times 8^3 = 8^9$

    So he wonders what would happen in the following situation: $8^{\#} \times 8^{\#} \times 8^{\#} = 8^1$

    a. What value does # have?
    b. What does $8^{\#}$ equal?

9. **(CA)** Here is a graph of the function $f(x)$. Answer the following questions about $f(x)$.



a. State the domain and range of $f(x)$.
b. Evaluate (by estimation) the following: (i) $f(4)$ (ii) $f(8)$ (iii) $f(6)$.
c. Solve for x in the following equations: (i) $f(x) = 3$ (ii) $f(x) = 1$ (iii) $f(x) = 0$.
d. The function is now moved using a translation vector of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Sketch the new function.

10. **(CI)** Evaluate (simplify as a number) the following:

a. (i) $4^2 + 4^1 + 4^0 + 4^{\frac{1}{2}}$ (ii) $\left(\frac{9}{16}\right)^{\frac{1}{2}} + \left(\frac{4}{9}\right)^{\frac{1}{2}}$ (iii) $\left(\frac{36}{25}\right)^{-\frac{1}{2}} + 9^{-\frac{1}{2}}$

11. **(CI)** Determine the intersection point of the following functions using algebraic methods.
(CA) Verify your solutions by graphing on your TI-84.

a. $2x - 5y = 12$ and $-4x + y = 12$
b. $2x + 5y = 12$ and $y = -3x - 8$
c. $y = 2^x$ and $y = 8 - 3(2^x)$

HOMEWORK PROBLEMS:

1.
2.
3.