IM2 Problem Set 5.3 - Working with Exponential Functions

BIG PICTURE of this UNIT:	- How can we analyze growth or decay patterns in data sets & contextual problems? - How can we algebraically & graphically summarize growth or decay patterns? - How can we compare & contrast linear and exponential models for growth and decay problems. - How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CA)** An exponential function has the form $f(x) = b^x$ where a is the **initial value**, b is the **growth factor.** Copy this table into your notes and
   a. decide if they can be used to model growth or decay
   b. determine the growth factor at which the change happens.

	Growth or Decay?	Growth/Decay Factor
$f(x) = 200(1.15)^x$		
$f(x) = 400(0.85)^x$		
$f(x) = 100(2)^x$		
$f(x) = 100(½)^x$		
$f(x) = 200(1.05)^x$		
$f(x) = 400(1.75)^x$		
$f(x) = 100(0.75)^x$		
$f(x) = 100(0.995)^x$		
$f(x) = 1{,}000(0.30)^x$		
$f(x) = 2500(1)^x$		

2. **(CI)** Use the exponent laws to write each expression with a single, simplified base. All exponents must be positive in your final answers.

   a. (i) $a^4 \times a^5 \times a^6$ (ii) $\frac{a^4}{a^{10}}$ (iii) $\frac{d}{d^{-5}}$ (iv) $(2a^2b^{-3})^2$
   b. (i) $a^4 \times a^{-5} \times a^{-3}$ (ii) $(3a^{-2}b^3)^{-2}$ (iii) $\frac{b^4 \times b^3}{b^2 \times a^{-2}}$ (iv) $\frac{(K^a)^b \cdot K^{ab}}{K^{4ab}}$

3. **(CI)** Evaluate (simplify as a number) the following:

   a. (i) -3^2 (ii) $(-3)^2$ (iii) -3^{-2} (iv) $(-3)^{-2}$ (v) $(3^{-2} + 3^{-1})^{-1}$
   b. (i) $\left(\frac{-2}{5}\right)^2$ (ii) $\left(\frac{-2}{5}\right)^{-2}$ (iii) $\left[\left(\frac{-2}{5}\right)^{-2}\right]^{-1}$ (iv) $-\left(\frac{-2}{5}\right)^2$ (v) $\left(\frac{-2}{5}\right)^3$

4. **(CI)** Difference Analysis of a Data Set. Mr S. gives you this data set and is asking you to analyze patterns in the data set in order to determine an equation in the form of $f(x) = mx + b$ for the data set.

x	-2	-1	0	1	2	3	4
$f(x)$	-1	2	5	8	11	14	17

a. Determine the "common difference" between each pair of terms (you do this by subtracting the successive y terms ==> difference = $y_2 - y_1$; $d = y_3 - y_2$; $d = y_4 - y_3$; etc …..
b. This value for the common difference is the ***slope*** or m in the equation. How can you use the data set to find the value for b?
c. Finally, what is the equation for this data set?

5. **(CA)** Ratio Analysis of a Data Set. Mr S. gives you this data set and is asking you to analyze patterns in the data set in order to determine an equation in the form of $f(x) = ab^x$ for the data set.

x	-2	-1	0	1	2	3	4
$f(x)$	$\frac{8}{9}$	$\frac{4}{3}$	2	3	4.5	6.75	10.125

a. Determine the "common ratio" between each pair of terms (you do this by dividing the successive y terms ==> ratio = $\frac{y_2}{y_1}$; ratio = $\frac{y_3}{y_2}$; r = $\frac{y_4}{y_3}$; etc …..
b. This value for the common ratio is the ***base*** or b in the equation. How can you use the data set to find the value for a?
c. Finally, what is the equation for this data set?

6. **(CA)** Graph the function $g(x) = 2^x$ on your calculator.

a. Determine the equation of the asymptote and the x- and y-intercept(s) - if they exist.
b. Sketch $g(x)$ into your notes.
c. Mr. R. is asking you to predict the appearance of the function, $h(x) = -2^{x+3} + 2$.

   i. Determine the x- and y-intercept(s) of $h(x)$, without using your calculator.
   ii. Predict where the asymptote of $h(x)$ might be. Explain your reasoning for this prediction.
   iii. Sketch $h(x)$.
   iv. Now graph $h(x)$ on your calculator and find the x- and y-intercepts and asymptote and compare to your prediction.
   v. Finally, state the domain and range of $h(x)$.

Part 2 - Skills/Concepts Application Problems

7. **(CA)** Ten grams of a chemical - Mathonium - is stored in a container. The amount of Mathonium present in the container can be modeled by $C(t) = 12.5(0.975)^t$, where C is the amount of Mathonium, in grams, and t is time in years from 2019.

   a. By considering only the equation, is the amount of Mathonium increasing or decreasing over time?
   b. Evaluate and interpret C(0) and C(100).
   c. Find the value of t where $C(t) = 10$. Round your final answer to the nearest year.

8. **(CA)** At the beginning of an experiment, there are 212 bacteria. The population of bacteria will double every 2 days. How many bacteria will be present in:

   a. 8 days b. 11 days c. 2 months d. 1 day e. 12 hours

9. **(CA)** A population of hamsters will triple every year. Initially, the population started with 10 hamsters.

   a. What will be the population of hamsters after 4 years? What assumptions are you making? Are these assumptions reasonable?
   b. How long will it take to get a population of 1500 hamsters?
   c. Mr. S would like to predict the number of hamsters present in 6 months. Explain how he could do this.

10. **(CA)** A colony of 1000 ants is growing at a rate of 15% every month.

   a. How many ants will be in the colony after 10 months? What assumptions are you making?
   b. How long will it take to get a population of 7500 ants?
   c. Mr. S would like to predict the number of ants present in one week. Explain how he could do this.

11. **(CI)** Mr D makes the following observation about exponents and exponent rules:

   (i) $4^1 \times 4^1 = 4^2$ (ii) $4^2 \times 4^2 = 4^4$ (iii) $4^3 \times 4^3 = 4^6$

   So he wonders what would happen in the following situation: $4^{\#} \times 4^{\#} = 4^1$

   a. What value does # have?
   b. What does $4^{\#}$ equal?