IM2 Problem Set 5.1 - Working with Exponential Functions

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. (CA) Here are four number patterns. Describe the pattern in each set and then predict the (i) the **next** 3 terms and then (ii) the 3 terms that **came before** the first term listed.
   a. Set 1 ⇒ {...., 15, 17, 19, 21, 23,....... }
   b. Set 2 ⇒ { ..., 20, 40, 80, 160, 320, ... }
   c. Set 3 ⇒ { ....., -14, -17, -20, -23, -26, ...}
   d. Set 4 ⇒ { ...., 8100, 2700, 900, 300, ... }

2. (CA) Consider the previous four number patterns. Is the number given part of the set? Show/explain why or why not.
   a. Is the number 2019 a part of Set 1? ⇒ {..., 15, 17, 19, 21, 23,....... }
   b. Is the number 512,000 part of Set 2? ⇒ { ..., 20, 40, 80, 160, 320, ... }
   c. Is the number 154 part of Set 3? ⇒ { ....., -14, -17, -20, -23, -26, ...}
   d. Is the number $\frac{1}{9}$ part of Set 4? ⇒ { ...., 8100, 2700, 900, 300, ... }

3. (CI) Using your knowledge of the "Exponent Laws", simplify each expression. All final answers should only have positive exponents.

   a. (i) $(2x^2)(4x^3y^2)$ (ii) $(-3a^2b)(6ab^4)$ (iii) $(7p^5q^2)(-3p^{-2}q^{-3})$
   b. (i) $(2x^3y)^3$ (ii) $2(x^3y)^3$ (iii) $(3ab^2)(2a^2b^3)^2$
   c. (i) $\frac{5x^3y^2}{10xy}$ (ii) $\frac{27a^4b^2}{18a^2b^4}$ (iii) $\frac{32(2x^2)^3y^2}{128x^4y^5}$

4. (CA) Using both DESMOS and your graphing calculator, graph the function $f(x) = 2^x$.

   a. Use your data table on the TI-84 and complete the following data table:

x	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$										

   b. Graph the line $y = 0$. Does the function $f(x) = 2^x$ ever go below $y = 0$?
   c. What is an asymptote?

5. **(CA)** Using both DESMOS and your graphing calculator, graph the function $f(x) = \left(\frac{1}{2}\right)^x$.

   a. Use your data table on the TI-84 and complete the following data table:

x	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$										

   b. Graph the line $y = 0$. Does the function $f(x) = \left(\frac{1}{2}\right)^x$ ever go below $y = 0$?
   c. What is an asymptote?

6. **(CA)** Graph the function $g(x) = -8(2^x) + 4$ on your calculator and on DESMOS. Find the x- and y-intercepts as well as the equation of the asymptote. Then sketch this graph in your notebooks, labeling the intercepts, the asymptote and include the equation of the function.

7. **(CI)** A relation is defined by the following description: To generate the numbers in this relation, the **starting number** will be 4. Every subsequent number is made by always **reducing the previous number** by a factor of 2. Create a table of values for this relation and then graph this relation. Predict an equation for this relation.

Part 2 - Skills/Concepts Application Problems

8. **(CA)** Since January 1st, 1980, the population of MaadiVille has grown according to the mathematical model $P(t) = 720500(1.022)^t$, where t is the number of years since 1980.

   a. Enter the equation into your TI-84 and look at your data table on the calculator. Now, set your windows so that you can see the function.
   b. Why would this curve be considered a "growth curve"?
   c. Evaluate and interpret (i) $P(40)$ and (ii) $P(-10)$
   d. Solve the equation $P(t) = 100{,}000$ using (i) your data table, and (ii) your graph.

9. **(CA)** Since January 1st, 2000, the occurrence of the disease called Mathitis has been changing according to the mathematical model $A(t) = 750(0.90)^t$, where t is the number of years since 2000.

   a. Enter the equation into your TI-84 and look at your data table on the calculator. Now, set your windows so that you can see the function.
   b. Why would this curve be considered a "decay curve"?
   c. Evaluate and interpret $P(19)$.
   d. Solve the equation $P(t) = 100$ using (i) your data table, and (ii) your graph.

10. **(CI)** Solve the following equations for x.

a. (i) $2x - 3 = 5$ (ii) $2^x - 3 = 5$
b. (i) $2(x + 1) - 13 = 3$ (ii) $2^{x+1} - 13 = 3$
c. (i) $2x + 5 = 3(2x) - 11$ (ii) $2^x + 5 = 3(2^x) - 11$

11. **(CI)** Evaluate the following numerical expressions.

a. i. $5^2 + 5^1 - 5^0$. ii. $(3 - 5)^3 - 2^{-1}$. iii. $(2^2 + 2^3)^2 + 3^2 - 4^0$
b. i. $(-2)^3 + 3^{-2}$. ii. $-2^3 + (2^2)^3 - 78^0$. iii. $\left(\frac{-2}{5}\right)^2 + \left(\frac{-2}{5}\right)^{-2}$

12. **(CA)** There is a well-known story of the man who invented chess. The local ruler was so pleased with the invention that he offered the inventor a great reward in gold. The inventor suggested an alternative reward: he would get one grain of rice on the first square of the chess board, two grains on the second square, four on the third, eight on the fourth, etc., doubling the number of grains each time. The ruler saw that this must be a much better deal for him, and accepted. The board has 64 squares.

a. If I only want to cover one row of the chess board (one row has 8 squares), how many grains of rice do I need?
b. How many total grains of rice did the ruler have to pay the inventor? Show your work.
c. If these grains of rice were lines up end to end, how far would the line go? (Assume each grain of rice is about 0.3cm in length.) Show your work.

The Legend of the Ambalappuzha Paal Payasam is an alternate version of the same story. Check it out!

HOMEWORK PROBLEMS:

13. From the Nelson 9 textbook, Chap 2.2 ⇒ starting on page 90, Q11,12

14. From the Nelson 9 textbook, Chap 2.3 ⇒ starting on page 96, Q910,11