1 The Tangent Ratio

What you'll learn

You'll learn to find the tangent of an angle and find missing measures using the tangent.

When am I ever going to use this?

Knowing how to use the tangent ratio can help you find unknown measures in triangles.

Word Wise

tangent

The industrial technology class plans to add a wheelchair ramp to the emergency exit of the auditorium as a class project. They know that the landing is 3 feet high and that the angle the ramp makes with the ground cannot be greater than 6°. What is the minimum distance from the landing that the ramp should start? *This problem will be solved in Example 1.*

Problems like the one above involve a right triangle and ratios. One ratio, called the **tangent**, compares the measure of the leg opposite an angle with the measure of the leg adjacent to that angle. The symbol for the tangent of angle A is tan A.

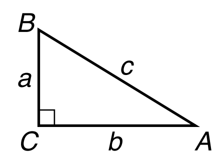
Tangent Ratio

Words: If A is an acute angle of a right triangle,

$$\tan A = \frac{\text{measure of the leg opposite } \angle A}{\text{measure of the leg adjacent to } \angle A}.$$

Symbols: $\tan A = \frac{a}{b}$

Model:



You can also use the symbol for tangent to write the tangent of an angle measure. The tangent of a 60° angle is written as tan 60°. If you know the measures of one leg and an acute angle of a right triangle, you can use the tangent ratio to solve for the measure of the other leg.

Example 1 APPLICATION

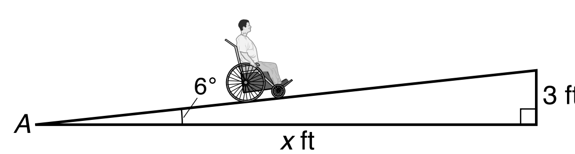
Construction **Solve the problem about the wheelchair ramp.**

First, draw a diagram.

$m\angle A = 6°$

adjacent leg = x feet

opposite leg = 3 feet



Now substitute these values into the definition of tangent.

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

$$\tan 6° = \frac{3}{x}$$

$$(\tan 6°)(x) = 3 \qquad \textit{Multiply each side by } x.$$

$$x = \frac{3}{\tan 6°} \qquad \textit{Divide each side by } \tan 6°.$$

3 [÷] 6 [TAN] [=] 28.54309336

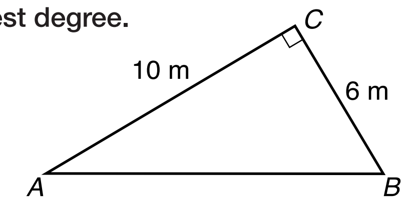
To the nearest tenth, the ramp should begin about 28.5 feet from the landing.

If your calculator does not have a [TAN] *key, you can use the table on the back cover of this booklet to estimate answers.*

You can use the TAN^{-1} function on your calculator to find the measure of an acute angle of a right triangle when you know the measures of the two legs.

Example

2 **Find the measure of $\angle A$ to the nearest degree.**



From the figure, you know the measures of the two legs. Use the definition of tangent.

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

$$\tan A = \frac{6}{10}$$

Now use your calculator to find the measure of $\angle A$.

6 $\boxed{\div}$ 10 $\boxed{=}$ $\boxed{\text{2nd}}$ [TAN^{-1}] 30.96375653

The measure of A is about 31°.

Study Hint

Technology To find TAN^{-1}, press the $\boxed{\text{2nd}}$ key and then the $\boxed{\text{TAN}}$ key.

CHECK FOR UNDERSTANDING

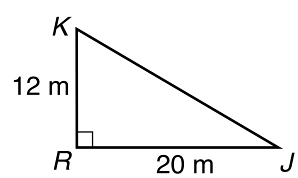
Communicating Mathematics

Read and study the lesson to answer each question.

1. ***Write*** a definition of the tangent ratio.
2. ***Tell*** how to use the tangent ratio to find the measure of a leg of a right triangle.
3. ***Tell*** how to find the measure of an angle in a right triangle when you know the measures of the two legs.

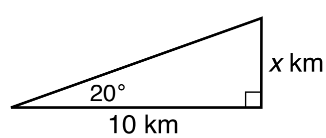
Guided Practice

Find each tangent to the nearest tenth. Find the measure of each angle to the nearest degree.



4. $\tan J$
5. $\tan K$
6. $m\angle J$
7. $m\angle K$

8. Find the value of x to the nearest tenth.

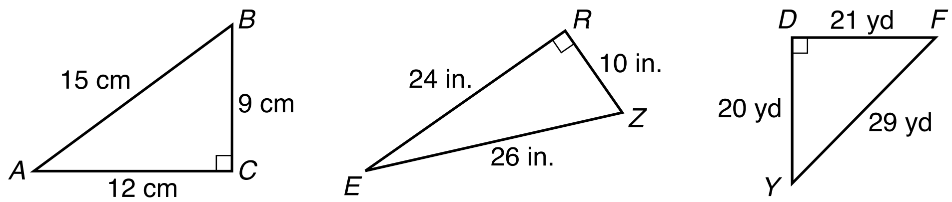


9. ***Measurement*** A guyline is fastened to a TV tower 50 feet above the ground and forms an angle of 65° with the tower. How far is it from the base of the tower to the point where the guyline is anchored into the ground? Round to the nearest foot.

EXERCISES

Practice

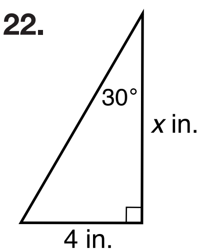
Complete each exercise using the information in the figures. Find each tangent to the nearest tenth. Find the measure of each angle to the nearest degree.



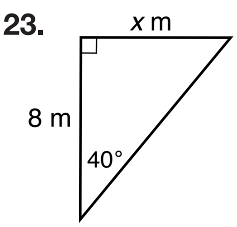
10. tan A	**11.** tan B	**12.** $m\angle A$	**13.** $m\angle B$
14. tan Z	**15.** tan E	**16.** $m\angle Z$	**17.** $m\angle E$
18. tan F	**19.** tan Y	**20.** $m\angle F$	**21.** $m\angle Y$

Find the value of *x* to the nearest tenth.

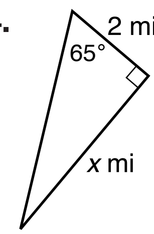
22.



23.



24.

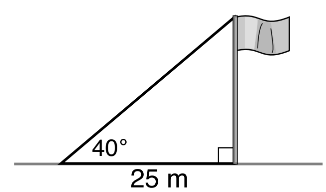


25. If the leg opposite the 53° angle in a right triangle is 4 inches long, how long is the other leg to the nearest tenth?

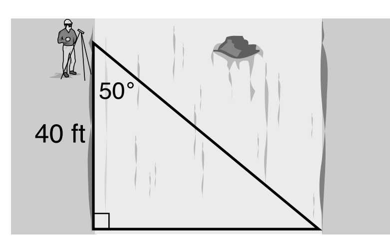
26. If the leg adjacent to a 29° angle in a right triangle is 9 feet long, what is the measure of the other leg to the nearest tenth?

Applications and Problem Solving

27. ***Measurement*** A flagpole casts a shadow 25 meters long when the angle of elevation of the Sun is 40°. How tall is the flagpole to the nearest meter?



28. ***Surveying*** A surveyor is finding the width of a river for a proposed bridge. A theodolite is used by the surveyor to measure angles. The distance from the surveyor to the proposed bridge site is 40 feet. The surveyor measures a 50° angle to the bridge site across the river. Find the length of the bridge to the nearest foot.



29. They are equal.

29. ***Critical Thinking*** In a right triangle, the tangent of one of the acute angles is 1. Describe how the measures of the two legs are related.

Name ______________________________ Date ______________

Study Guide

The Tangent Ratio

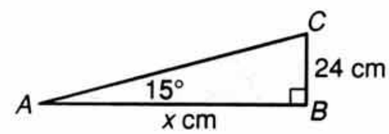
The tangent ratio compares the measure of the leg opposite an angle with the measure of the leg adjacent to that angle.

If A is an acute angle of a right triangle, then

$$\tan A = \frac{\text{measure of the leg opposite } \angle A}{\text{measure of the leg adjacent to } \angle A}$$

Example 1 Use the tangent ratio to find the value of x.

From the diagram: $m\angle A = 15°$
adjacent leg = x cm
opposite leg = 24 cm



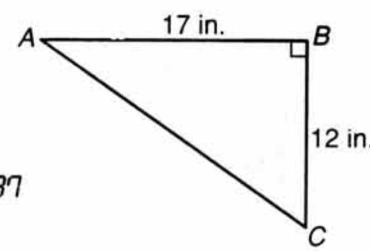
$\tan 15° = \frac{24}{x}$ ← opposite leg / ← adjacent leg

$(\tan 15°)(x) = 24$

$x = \frac{24}{\tan 15°}$

Use a calculator: 24 [÷] 15 [TAN] [=] 89.56921938
$x \approx 89.6$

Example 2 Find the measure of $\angle A$.



$\tan A = \frac{12}{17}$ ← opposite leg / ← adjacent leg

Use a calculator:
12 [÷] 17 [=] [2nd] [TAN^{-1}] 35.21759297
$m\angle A \approx 35°$

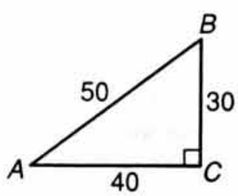
Complete each exercise using the information in the figure. Find angle measures to the nearest degree.

1. $\tan A =$ _____

$\tan B$ _____

$m\angle A =$ _____

$m\angle B =$ _____

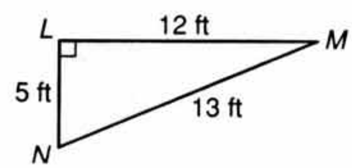


2. $\tan M =$ _____

$\tan N =$ _____

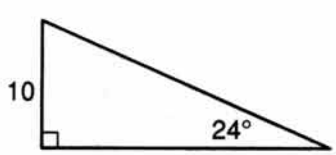
$m\angle M =$ _____

$m\angle N =$ _____



Find the value of x to the nearest tenth or degree.

3.



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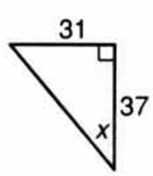

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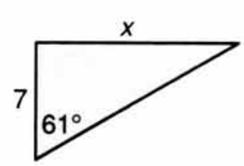

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Practice

The Tangent Ratio

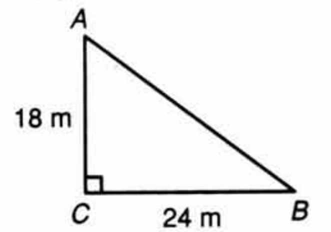
Use the figures at the right for Exercises 1-8. Write the ratios in simplest form. Find angle measures to the nearest degree.

1. Find tan A.

2. Find $m\angle A$.

3. Find tan B.

4. Find $m\angle B$.

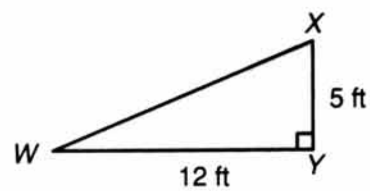


5. Find tan W.

6. Find $m\angle W$.

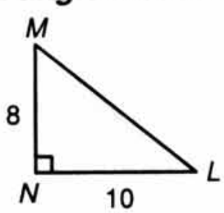
7. Find tan X.

8. Find $m\angle X$.



Complete each exercise using the information in the figure. Find angle measures to the nearest degree.

9.



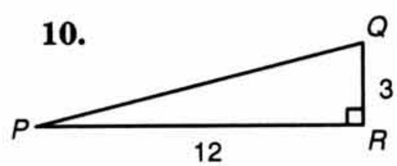
tan L = ?

$m\angle L$ = ?

tan M = ?

$m\angle M$ = ?

10.



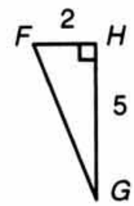
tan P = ?

$m\angle P$ = ?

tan Q = ?

$m\angle Q$ = ?

11.



tan F = ?

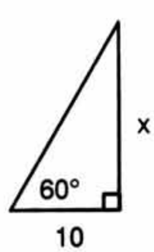
$m\angle F$ = ?

tan G = ?

$m\angle G$ = ?

Find the value of x to the nearest tenth or to the nearest degree.

12.



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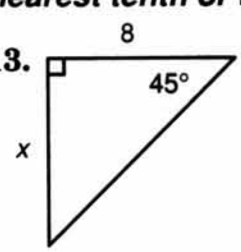
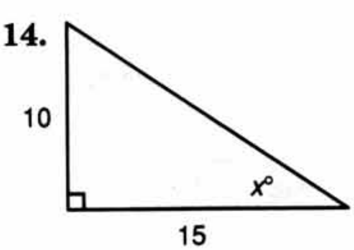


14.



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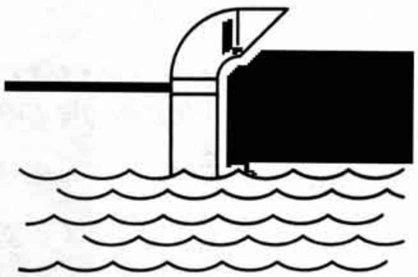
Enrichment

On a Clear Day



If you've ever watched a submarine movie, you've probably seen the captain of the submarine look through a periscope. Have you ever wondered how far the captain could see?

The equation that approximates the distance that someone can see is $d = 1.5\sqrt{h}$ where d is the distance in miles and h is the height in feet of the observer above the surface.

When using this formula, remember that it gives answers only for ideal conditions. Viewing conditions on Earth are typically less than ideal.

Use the equation $d = 1.5\sqrt{h}$ and your calculator to answer the following questions. When necessary, round your answers to the nearest hundredth.

1. A submarine captain is looking through a periscope that is 1 foot above the water. How far is the captain able to see?

2. An airplane is flying at an altitude of 6.5 miles. When looking out the window, how far would an observer be able to see? (5,280 feet =1 mile)

3. The Sears Tower in Chicago is 1,464 feet tall. If the observation deck is located at a height of 1,350 feet above ground, how far would a person on the observation deck be able to see?

4. A sailor is standing in the crow's nest of a sailing ship. The crow's nest is an observation area located high above the ship on a mast. The sailor in the crow's nest is 120 feet above the surface of the water. How far is the sailor able to see?

5. Using a periscope, a submarine captain sights a rowboat that is floating at a distance of 1.84 miles from the ship. How far is the periscope above water?

6. When looking out the window of an airplane, an observer is able to see a distance of 150 miles. At what height is the airplane flying?