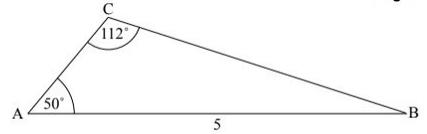


# Math SL PROBLEM SET 99

## Section A (Skills/Concepts Consolidation)

1. (CA) The following diagram shows a triangle ABC, where  $AB = 5$  cm,  $\angle CAB = 50^\circ$  and  $\angle ACB = 112^\circ$   
(a) Find BC. (b) Find the area of triangle ABC.



2. (CA) Let  $f(x) = \frac{6x^2 - 4}{e^x}$  for  $0 \leq x \leq 7$ .  
a. Find the  $x$ -intercept of the graph of  $f$ .  
b. The graph of  $f$  has a maximum at the point A. Write down the coordinates of A.  
c. Sketch the graph of  $f$ .

3. (CA) Let  $\vec{AB} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ . (a) Find  $|\vec{AB}|$ . (b) Let  $\vec{AC} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$ . Find  $\angle BAC$ .

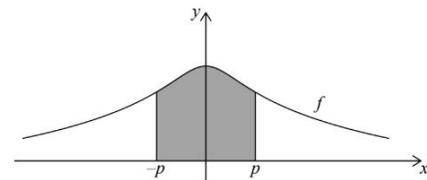
4. (CA) A discrete random variable  $X$  has the following probability distribution.  
a. Find the value of  $k$ .  
b. Write down  $P(X = 2)$ .  
c. Find  $P(X = 2 | X > 0)$ .

$X$	0	1	2	3
$P(X=x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

5. (CA) Let  $f(x) = 6 - \ln(x^2 + 2)$ , for  $x \in \mathbb{R}$ . The graph of  $f$  passes through the point  $(p, 4)$ , where  $p > 0$ .  
a. Find the value of  $p$ .

The following diagram shows part of the graph of  $f$ .

- b. The region enclosed by the graph of  $f$ , the  $x$ -axis and the lines  $x = -p$  and  $x = p$  is rotated  $360^\circ$  about the  $x$ -axis. Find the volume of the solid formed.



6. (CA) In the expansion of  $ax^3(2 + ax)^{11}$ , the coefficient of the term in  $x^5$  is 11880. Find the value of  $a$ .
7. (CA) The heights of adult males in a country are normally distributed with a mean of 180 cm and a standard deviation of  $\sigma$  cm. 17% of these men are shorter than 168 cm. Eighty percent of them have heights between  $(192 - h)$  cm and 192 cm. Find the value of  $h$ .

## Section B (Skills/Concepts Practice)

8. (CA) A particle P moves in a straight line for five seconds. Its acceleration at time  $t$  is given by the equation  $a(t) = 3t^2 - 14t + 8$ , for  $0 \leq t \leq 5$ .  
a. Write down the values of  $t$  when  $a = 0$ .  
b. Hence or otherwise, find all possible values of  $t$  for which the velocity of P is decreasing.  
When  $t = 0$ , the velocity of P is  $3 \text{ m s}^{-1}$ .  
c. Find an expression for the velocity of P at time  $t$ .  
d. Find the total distance travelled by P when its velocity is increasing.

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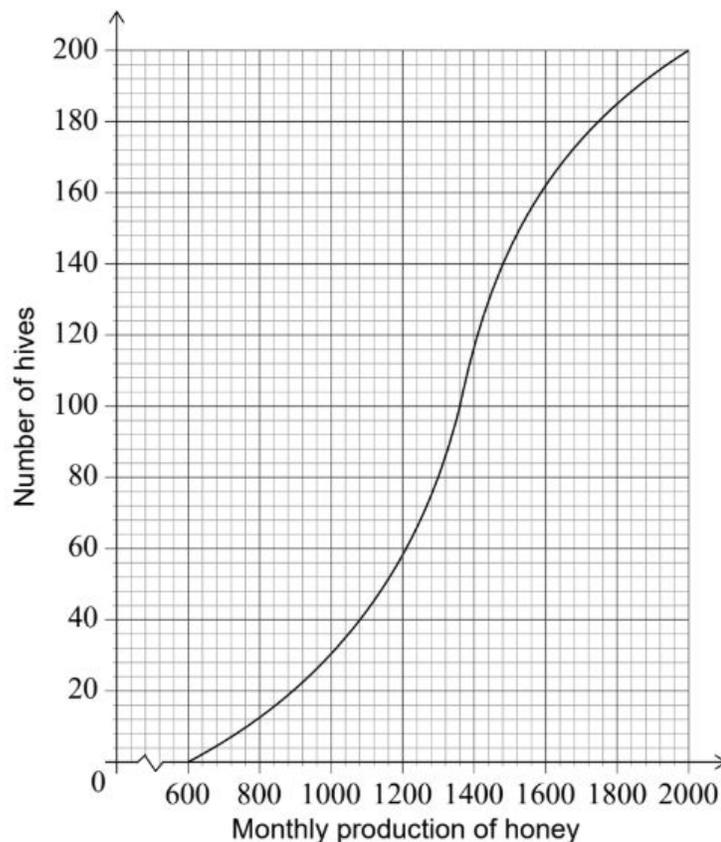
9. (CA) Adam is a beekeeper who collected data about monthly honey production in his bee hives. The data for six of his hives is shown in the following table.

<b>Number of bees (<math>N</math>)</b>	190	220	250	285	305	320
<b>Monthly honey production in grams (<math>P</math>)</b>	900	1100	1200	1500	1700	1800

The relationship between the variables is modelled by the regression line with equation  $P = aN + b$ .

- Write down the value of  $a$  and of  $b$ .
- Use this regression line to estimate the monthly honey production from a hive that has 270 bees.

Adam has 200 hives in total. He collects data on the monthly honey production of all the hives. This data is shown in the following cumulative frequency graph.



Adam's hives are labelled as low, regular or high production, as defined in the following table.

<b>Type of hive</b>	low	regular	high
<b>Monthly honey production in grams (<math>P</math>)</b>	$P \leq 1080$	$1080 < P \leq k$	$P > k$

- Write down the number of low production hives.

Adam knows that 128 of his hives have a regular production.

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- d. Find (i) the value of  $k$  ; (ii) the number of hives that have a high production.
- e. Adam decides to increase the number of bees in each low production hive. Research suggests that there is a probability of 0.75 that a low production hive becomes a regular production hive. Calculate the probability that 30 low production hives become regular production hives.

10. (CA) Let  $f(x) = x + a \sin(x - \frac{\pi}{2}) + a$ , for  $x \geq 0$ .

- a. Show that  $f(2\pi) = 2\pi$ .

The graph of  $f$  passes through the origin. Let  $P_k$  be any point on the graph of  $f$  with  $x$ -coordinate  $2k\pi$ , where  $k \in \mathbb{N}$ . A straight line  $L$  passes through all the points  $P_k$ .

- b. (i) Find the coordinates of  $P_0$  and of  $P_1$ .  
 (ii) Find the equation of  $L$ .
- c. Show that the distance between the  $x$ -coordinates of  $P_k$  and  $P_{k+1}$  is  $2\pi$ .

Diagram 1 shows a saw. The length of the toothed edge is the distance  $AB$ . The toothed edge of the saw can be modelled using the graph of  $f$  and the line  $L$ .

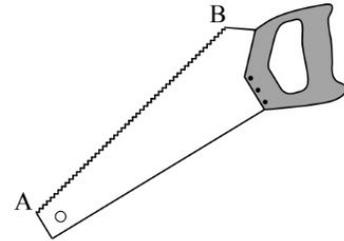


Diagram 2 represents this model. The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of  $f$  and the line  $L$ , between  $P_k$  and  $P_{k+1}$ .

- d. A saw has a toothed edge which is 300 mm long. Find the number of complete teeth on this saw.

