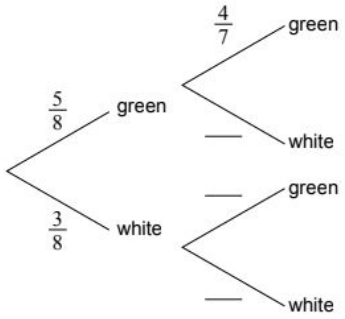
Math SL PROBLEM SET 98

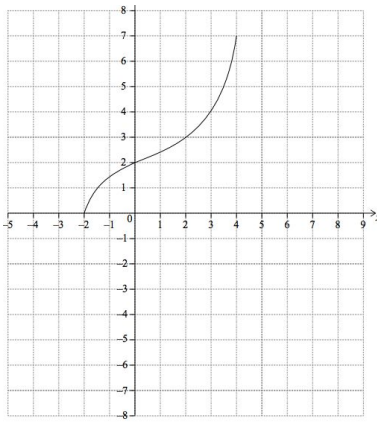
Section A (Short Response)

1. A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.
   a. Complete the following tree diagram.
   b. Find the probability that exactly one of the selected balls is green.

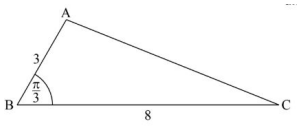


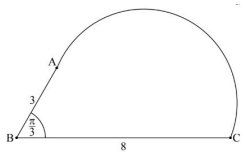
2. In an arithmetic sequence, the first term is 8 and the second term is 5.
   a. Find the common difference.
   b. Find the tenth term.
   c. Find the sum of the first ten terms.

3. The diagram shows the graph of a function f, with domain $-2 \leq x \leq 4$. The points (-2 , 0) and (4 , 7) lie on the graph of f.
   a. Write down the range of f.
   b. Write down (i) $f(2)$; (ii) $f^{-1}(2)$.
   c. On the grid opposite, sketch the graph of f^{-1}.



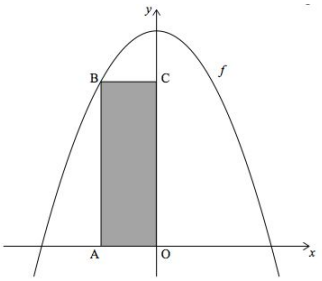
4. The following diagram shows triangle ABC, with AB = 3 cm , BC = 8 cm , and $\angle ABC = \frac{\pi}{3}$.
   a. Show that AC = 7 cm .
   b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle. Find the exact perimeter of this shape.





5. Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \boldsymbol{R}$, where b is a constant.
   a. Find $(g \circ f)(x)$.
   b. Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b.

6. Let $f(x) = 15 - x^2$, for $x \in \boldsymbol{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis. Find the x-coordinate of A that gives the maximum area of OABC.



7. Consider $f(x) = \log_k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$. The equation $f(x) = 2$ has exactly one solution. Find the value of k.

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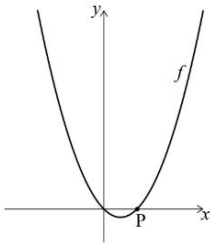
Section B (Extended Response)

8. Let $f(x) = x^2 - x$, for $x \in \boldsymbol{R}$. The following diagram shows part of the graph of f. The graph of f crosses the x-axis at the origin and at the point P (1 , 0).

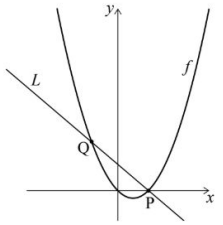


a. Show that $f'(1) = 1$.

The line L is the normal to the graph of f at P.

b. Find the equation of L in the form $y = ax + b$.

The line L intersects the graph of f at another point Q, as shown in the following diagram.



c. Find the x-coordinate of Q.

d. Find the area of the region enclosed by the graph of f and the line L.

9. A line L passes through points A (-3 , 4 , 2) and B (-1 , 3 , 3).

a. (i) Show that $\vec{AB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

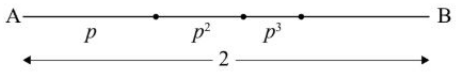
(ii) Find a vector equation for L.

The line L also passes through the point C (3 , 1 , p).

b. Find the value of p.

c. The point D has coordinates (q^2 , 0 , q). Given that DC is perpendicular to L, find the possible values of q.

10. The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments. The length of the line segments are p cm , p^2 cm , p^3 cm , ... , where $0 < p < 1$.



a. Show that $p = \frac{2}{3}$.

b. The following diagram shows [CD], with length b cm , where $b > 1$. Squares with side lengths k cm , k^2 cm , k^3 cm , ... , where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.

The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.