Math SL PROBLEM SET 97

1. (CI) Let $f(x) = x^2 + x - 6$.
    a. Write down the y-intercept of the graph of f.
    b. Solve $f(x) = 0$.
    c. Sketch the graph of f, for $-4 \le x \le 3$.

2. (CI) In an arithmetic sequence, the first term is 2 and the second term is 5.
    a. Find the common difference.
    b. Find the eighth term.
    c. Find the sum of the first eight terms of the sequence.

3. The following diagram shows a board which is divided into three regions A, B and C.

    A game consists of a contestant throwing one dart at the board. The probability of hitting each region is given in the following table.

Region	A	B	C
Probability	$\dfrac{5}{20}$	$\dfrac{4}{20}$	$\dfrac{1}{20}$
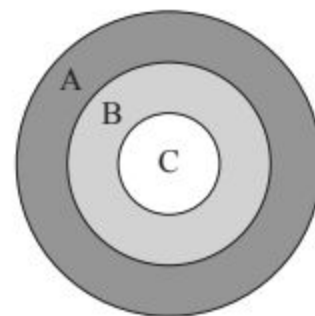
    a. Find the probability that the dart does **not** hit the board.

    The contestant scores points as shown in the following table.

| Region	A	B	C	Does not hit the board
Points	0	q	10	−3

    b. Given that the game is fair, find the value of q.

4. Given the expression $3\ln 2 - \ln 4$;
    a. write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbf{R}$.
   b. Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$.

5. Let $f(x) = P + \frac{9}{x-Q}$; $x \neq Q$. The line $x = 3$ is a vertical asymptote to the graph of f. The graph of f has a y-intercept at $(0, 4)$.

    a. Write down the value of Q.
    b. Find the value of P.
    c. Write down the equation of the horizontal asymptote of f.

6. The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$ for $0 \leq x \leq 4$, and the line $x = 4$. Let R be the region enclosed by the graph of f, the x-axis and the line $x = 4$. Find the area of R.
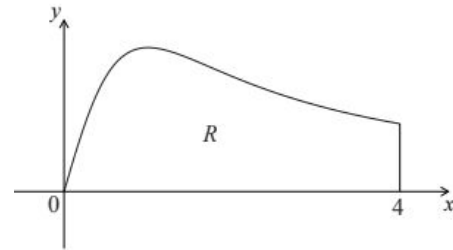
7. The following diagram shows triangle ABC. Let $\overrightarrow{AB} \cdot \overrightarrow{AC} = -5\sqrt{3}$ and let $|\overrightarrow{AB}||\overrightarrow{AC}| = 10$. Find the area of triangle ABC.
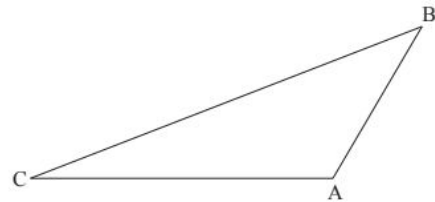
Section B (Skills/Concepts Practice)

8. Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle. The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car. The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle. This information is represented by the following tree diagram.

    a. Find the value of p.
    b. Find the probability that Adam will travel by car and be late for school.
    c. Find the probability that Adam will be late for school.
    d. Given that Adam is late for school, find the probability that he travelled by car.
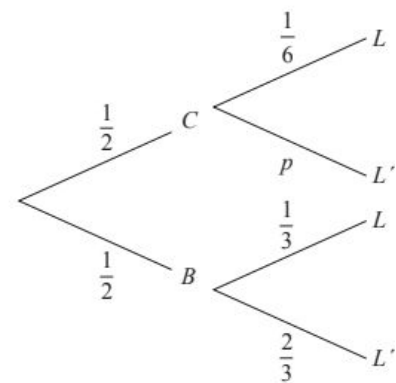
Adam will go to school three times next week.

    e. Find the probability that Adam will be late exactly once.

9. The following diagram shows the graph of a function f. There is a local minimum point at A, where $x > 0$. The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

   a. Find the x-coordinate of A.
   b. The y-intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$.

   The graph of a function g is obtained by reflecting the graph of f in the y-axis, followed by a translation of $\binom{m}{n}$.
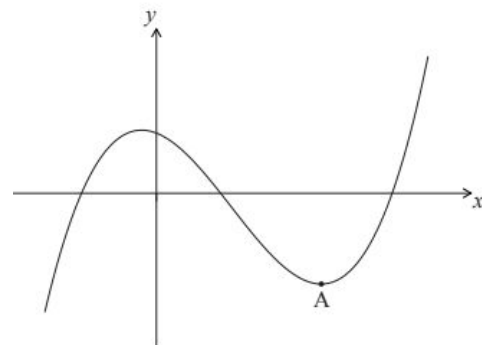
   c. Find the x-coordinate of the local minimum point on the graph of g.

10. Let L_x be a family of lines with equation given by $r = \begin{pmatrix} x \\ 2 \\ x \end{pmatrix} + t \begin{pmatrix} x^2 \\ -2 \end{pmatrix}$ where $x > 0$.

   a. Write down the equation of L_1.

   A line, L_a, crosses the y-axis at a point P.

   b. Show that P has coordinates $\left(0, \frac{4}{a}\right)$.

   The line L_a crosses the x-axis at $Q(2a,0)$. Let $d = PQ^2$.

   c. Show that $d = 4a^2 + \frac{16}{a^2}$.
   d. There is a minimum value for d. Find the value of a that gives this minimum value.