Math SL PROBLEM SET 96

Section A (Skills/Concepts Consolidation)

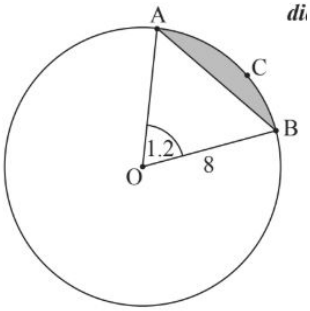
1. **(CA)** Let $f(x) = 2x + 3$ and let $g(x) = x^3$. Find $(f \circ g)(x)$ and hence solve $(f \circ g)(x) = 0$.

2. **(CA)** The following table shows the Diploma score x and the university entrance exam mark y for seven IB Diploma students. The relationship can be modelled by the regression line with equation $y = ax + b$.

Diploma score (x)	28	30	27	31	32	25	27
University entrance mark (y)	73.9	78.1	70.2	82.2	85.5	62.7	69.4

   a. Find the correlation coefficient.
   b. Write down the value of a and of b.
   c. Rita scored 26 in her IB Diploma. Use your regression line to estimate Rita's university entrance mark.

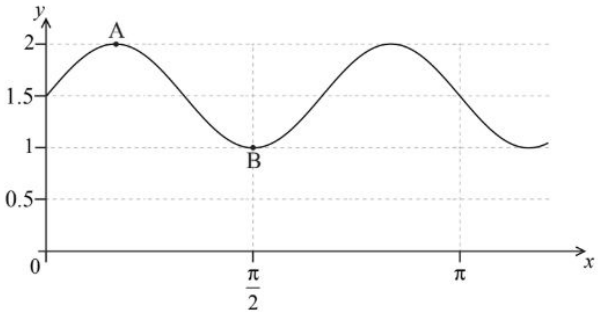
3. **(CA)** The following diagram shows a circle with a center O and a radius of 8 cm. The points A,B and C are on the circumference of the circle and $\angle AOB = 1.2$ radians.



   a. Find the length of arc ACB.
   b. Find AB.
   c. Hence, find the perimeter of the shaded figure.

4. **(CA)** Let $f(x) = -x^4 + 2x^3 - 1$ for $0 \leq x \leq 2$.
   a. Sketch the graph of f on the given domain.
   b. Solve $f(x) = 0$.
   c. The region enclosed by the graph of f and the x-axis is rotated 360 about the x-axis. Find the volume of the solid formed.

5. **(CA)** The following diagram shows part of the graph of $y = p \sin(qx) + r$. The point $A\left(\frac{\pi}{6}, 2\right)$ is a maximum point and the point $B\left(\frac{\pi}{2}, 1\right)$ is a minimum point. Find the values of p, q and r.



6. **(CA)** Consider the expansion of $\left(\frac{x^3}{2} + \frac{p}{x}\right)^8$. The constant term is 5103. Find the possible values of p.

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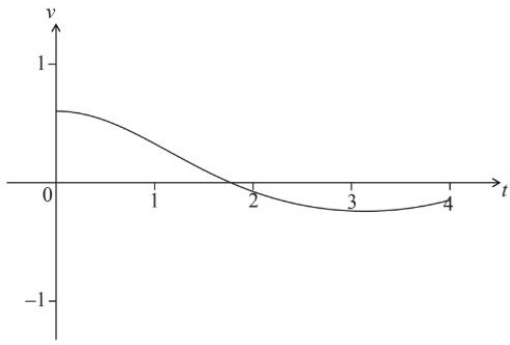
7. **(CA)** A particle starts from point A and moves along a straight line. Its velocity, v ms^{-1}, after t seconds is given by $v(t) = e^{\frac{1}{2}\cos(t)} - 1$, for $0 \le t \le 4$. The particle is at rest when t = $\frac{\pi}{2}$. The following diagram shows the graph of v.



   a. Find the distance travelled by the particle for $0 \le t \le \frac{\pi}{2}$.
   b. Explain why the particle passes through point A again.

Section B (Skills/Concepts Practice)

8. **(CA)** The first two terms of a geometric sequence, u_n, are $u_1 = 4$ and $u_2 = 4.2$.
   a. Find the common ratio and hence or otherwise, find u_5.

   Another sequence, v_n, is defined by $v_n = an^k$, where $a,k \in \boldsymbol{R}$ and $n \in \boldsymbol{Z}^+$, such that $v_1 = 0.05$ and $v_2 = 0.25$.
   b. Find the value of a and the value of k.
   c. Find the smallest value of n for which $v_n > u_n$.

9. **(CA)** The weights of fish in a lake are normally distributed with a mean of 760 g and a standard deviation of σ. It is known that 78.87% of the fish have weights between 705 g and 815 g.

   a. (i) Write down the probability that a fish weighs for than 760 g.
      (ii) Find the probability that a fish weighs less than 815 g.

   b. (i) Write down the standardized value for 815 g.
      (ii) Hence, or otherwise, find σ.

   A fishing contest takes place in the lake. Small fish, called tiddlers, are thrown back into the lake. The maximum weight of a tiddler is 1.5 standard deviations below the mean.

   c. Find the maximum weight of a tiddler.

   d. A fish is caught at random. Find the probability that it is a tiddler.

   e. Twenty-five percent of fish in the lake are salmon. Ten percent of the salmon are tiddlers. Given that a fish caught at random is a tiddler, find the probability that it is a salmon.

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10. **(CA)** The following cumulative frequency graph (on the next page) shows the monthly income, I dollars, of 2000 families.

a. Find the median monthly income.

b. (i) Write down the number of families who have a monthly income of $2000 or less.
(ii) Find the number of families who have a monthly income of more than 4000 dollars.

The 2000 families live in two different types of housing. The following table gives information about the number of families living in each type of housing and their monthly income I .

	$1000 < I \leq 2000$	$2000 < I \leq 4000$	$4000 < I \leq 5000$
Apartment	436	765	28
Villa	64	p	122

c. Find the value of p .

d. A family is chosen at random.

(i) Find the probability that this family lives in an apartment.
(ii) Find the probability that this family lives in an apartment, given that its monthly income is greater than 4000 dollars.

e. Estimate the mean monthly income for families living in a villa.

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