# Math SL PROBLEM SET 94

**Section A (Skills/Concepts Consolidation)**

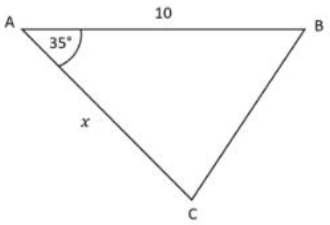
1. **(CI)** Let $f(x) = x^2 + 1$ and let $g(x) = \sqrt{x-3}$.
   a. Find $g^{-1}(x)$ b. Find $f \circ g\,(28)$ c. Write down the range of $f(x)$ and of $g^{-1}(x)$

2. **(CA)** A data set has a mean of 15 and a standard deviation of 4.
   a. Each value in the data set has 5 added to it. Write down the value of the new mean and the new standard deviation
   b. Each value in the original data set is multiplied by 5. Write down the value of the new mean and find the value of the new variance.

3. **(CI)** A discrete random variable $X$ has the following probability distribution. Find the value of $k$ and hence find $E(X)$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{6}{20}$ | $k$ | $\frac{2}{20}$ | $\frac{3}{20}$ | $\frac{5}{20}$ |

4. **(CA)** The following diagram shows $\Delta ABC$. The area of $\Delta ABC$ is 22 cm$^3$.
   a. Find $x$. b. Find BC. c. Find $\angle ACB$.




5. **(CI)** An arithmetic sequence has $u_1 = \log_k(ab)$ and $u_2 = \log_k(b)$ where $k > 1$ and $a, b > 0$.
   a. Show that $d = -\log_k(a)$
   b. Let $a = k^4$ and let $b = k^{16}$. Find the value of $\sum_{n=1}^{15} u_n$

6. **(CA)** In a geometric sequence $u_2 = 6$ and $u_5 = 20.25$.
   a. Find the value of $r$.
   b. Find $u_1$.
   c. Find the greatest value of $n$ such that $u_n < 200$.

7. **(CI)** Let $f(x) = \frac{2}{x-a} + b$, for $x \neq a$. The line $x = 1$ is a vertical asymptote to the graph of $f$. The graph passes through the point (0,3)
   a. Write down the value of $a$.
   b. Find the value of $b$.
   c. Find $\lim_{x \to \infty} f(x)$.

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**Section B (Skills/Concepts Practice)**

8. **(CA)** The heights of the flowers in a flower bed are normally distributed with a mean of 43 cm and a standard deviation of 6 cm. Flowers are classified as tall flowers if they have a height that is more than 48 cm.

   a. A flower is selected at random.Find the probability that this flower is a tall flower.
   b. Given that this flower is tall, find the probability that it is taller than 55 cm.
   c. Two flowers are selected at random. Find the probability that they are both tall.

   Five hundred flowers are selected at random.

   d. Find the expected value of these flowers that are tall.
   e. Find the probability that at least 100 of these flowers are tall.

9. **(CI)** Consider a function, $f$. The line $L_1$ with equation $y = 2x - 1$ is a tangent to the graph of $f$ when $x = 3$.

   a. (i) Write down $f'(3)$. (ii) Find $f(3)$.
   b. Let $g(x) = f(x^2 - 1)$ and $P$ be the point on the graph of $g$ where $x = 2$. Show that the graph of $g$ has a gradient of 8 at $P$.
   c. Let $L_2$ be the tangent to the graph of $g$ at $P$. The line $L_1$ intersects $L_2$ at the point $Q$. Find the $y$-coordinate of Q.

10. **(CA)** The diagram shows a parallelogram ABCD. The coordinates are A(2,3,4), B(8,5,7) and D(4,7,8).

    a. (i) Show that $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$. (ii) Find $\overrightarrow{AD}$. (iii) Hence, show that $\overrightarrow{AC} = \begin{pmatrix} 8 \\ 6 \\ 7 \end{pmatrix}$

    b. Find the coordinates of C.

    c. (i) Find $\overrightarrow{AB} \cdot \overrightarrow{AD}$
    (ii) Hence, find $\angle$BAD.

    d. Hence, or otherwise, find the area parallelogram ABCD.