Math SL PROBLEM SET 93

1. **(SP5.6)** **(CA)** The following probabilities involving events A and B are given:

   $$P(A`) = \tfrac{1}{4}, P(B|A) = \tfrac{2}{3} \text{ and } P(B`|A`) = \tfrac{3}{5}.$$

   a. Complete a tree diagram.
   b. Find the following probabilities:
      i. $P(A \cap B)$ ii. $P(B)$ iii. $P(A|B)$
   c. Prepare a Venn diagram for the events A and B
   d. Are the events independent or dependent? Show/explain why/why not.

2. **(CA6.3)** **(CI)** Consider the function $g(x) = x \ln(x^2)$.

   a. Find $g`(x)$ and $g``(x)$.
   b. Does the graph of g have an inflexion point. Explain.

3. **(A1.3)** **(CA)** Find the coefficient of x^4 in the expansion of $(x - 1)^2 (2x + 1)^4$.

4. **(CA6.3)** **(CI/CA)** The graph of the equation $y = e^x (1 - x^2)$ has two points of inflexion. Find the equations of the first and second derivatives and hence, find the coordinates of each of the inflection points. Then use a calculator to approximate the x- and y-coordinates to three significant figures.

5. **(V4.2)** **(CI)** Find the possible value(s) of a if:

   a. The cosine of the angle between $v = a\,i + 2\,j + k$ and $w = 3\,j - 3\,k$ is $\frac{\sqrt{3}}{6}$.
   b. The cosine of the angle between $v = 4\,i + 3\,j$ and $w = -i + 2\,j + a\,k$ is $\frac{2}{25}$.

6. **(CA6.3)** **(CA)** A rectangular box has height h cm, width x cm and length $2x$ cm. It is designed to have a volume equal to 1 litre (1000 cm^3).

   a. Show that $h = \frac{500}{x^2}$ cm.
   b. Find an expression for the total surface area, S cm^2 , of the box in terms of x.
   c. Find the dimensions of the box that produces a minimum surface area.

7. **(CA6.1; CA6.2) (CI)** Find the equation of the line that is tangent to the graph of $y = x \tan x$ at the point where $x = \frac{\pi}{4}$.

8. **(V4.2) (CI)** ABCD is a rectangle such that A(2, 1, 4), B(6, 5, 2), and C(2, 8, 3).

   a. Find the coordinates of point D.
   b. Find the cosine of the acute angle between the diagonals of the rectangle.

9. **(V4.3) (CA)** In this question, distance is in kilometres and time is in hours. A small drone (remote controlled aircraft) is moving at a constant height with a speed of 15 kmh^{-1} in the direction of $\begin{pmatrix} 7 \\ 24 \\ 0 \end{pmatrix}$. At time $t = 0$, the drone is at point P with coordinates (0, 0, 8).

   a. Show that the position vector, r_1, of the drone at time t is given by

   $$r_1 = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix} + t \begin{pmatrix} 4.2 \\ 14.4 \\ 0 \end{pmatrix}$$

   At time $t = 0$, a second drone flies to intercept the first drone (to connect together for a practice recovery). The position vector of this second drone, r, at time t is given by

   $$r_2 = \begin{pmatrix} 36.8 \\ 85.6 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5 \\ -7 \\ 2 \end{pmatrix}.$$

   b. (i) Write down the coordinates of the starting position of the second drone.

   (ii) Find the speed of the second drone.

   c. The second drone reaches the first drone at point Q.

   (i) Find the time it takes the second drone to reach the first drone.

   (ii) Find the coordinates of Q.