Math SL PROBLEM SET 92

Section A (Skills/Concepts Consolidation)

1. **(CA6.5)** The function $f(x)$ is given as $f(x) = \sqrt{\cos(x)}$.

   (Cirrito 22.7, p766; Cirrito 20.1, p643)

   a. **(CI)** Find the equation of the line tangent to f at $x = \frac{\pi}{3}$.
   b. **(CA)** The region R is bounded by $f(x) = \sqrt{\cos(x)}$ and the x and y axes. Find the volume of the solid created by revolving the region R 360° about the x-axis.

2. **(SP5.9) (CA)** The weights of a certain animal are normally distributed with a mean of 36.4 kg and a standard deviation of 4.7 kg. Find the probability that when one of these animals is chosen at random it will have a weight that is: (a) 40.0 kg or less; (b) more than 45.0 kg; (c) between 32.0 kg and 41.0 kg. **(Cirrito 17.2, p557)**

3. **(CA6.5; CA6.6) (CI)** The velocity meters/sec of an object moving along a line is given by the equation $v(t) = t^3 - 8$.

   (Cirrito 22.6, p762)

   a. Given that when $t = 2$ seconds the displacement of the object is 8 meters, then what is the displacement of the object at $t = 3$ seconds?
   b. What is the total distance travelled by the object from $t = 0$ to $t = 3$ seconds?

4. **(CA6.5) (CI)** For the function $f(x) = x^3 - 4x^2 + x + 6$, it is known that one root of $f(x)$ is at $x = -1$. Hence, find the **exact** area of the region bounded by the graph of $f(x)$ and the x-axis.
   (Cirrito 22.5, p758)

5. **(SP5.9) (CA)** Adult male customers for t-shirts have chest measurements which may be modelled by a normal distribution with mean 101 cm and standard deviation 5 cm. T-shirts to fit customers with chest measurements less than 98 cm are classified as 'small'. Find the median chest measurement of customers requiring 'small' t-shirts. **(Cirrito 17.2, p557)**

Section B (Skills/Concepts Practice)

6. **(CA6.5) (CI)** The region bounded by the graphs of $y = \frac{1}{2}x + 2$, the y-axis, the x-axis and the vertical line $x = m$ has an area of exactly 45 square units. Find the value of m.

   (Cirrito 22.5, p758)

7. **(CA6.2)** **(CI)** Find the derivatives of the following functions:

    a. $g(x) = x^2 \ln x$ b. $h(x) = e^x \sin 2x$ c. $k(x) = \frac{x^3 - 2x + 5}{x - 1}$

8. **(CA6.4)** **(CI)** Evaluate each definite integral:

    a. $\displaystyle\int_0^1 \left(4x - \sqrt{2x}\right) dx$ b. $\displaystyle\int_0^{2\pi} 3 \sin\left(\tfrac{x}{2}\right) dx$

9. **(SP5.6; SP5.8)** **(CA)** A bag contains 10 balls. 6 of the balls are green and 4 of the balls are red. A ball is selected at random, its colour is noted, and then it is replaced.

    a. What is the probability that the ball was green?

A total of 8 selections takes place. After each time, the ball is replaced.

    b. What is the probability that a green ball was selected on each of the 8 occasions?
    c. What is the probability that, in any order, a green ball was selected on four occasions, and a red ball was selected on four occasions?
    d. What is the probability that a red ball was selected at least twice?

A game is played as follows: If a green ball is drawn then a score of -1 is awarded. If a red ball is drawn then a score of $+1$ is awarded. A total of 8 selections takes place.

    e. (i) What is the probability of getting an odd score?
    (ii) What is the probability that 0 was scored in total?
    (iii) Five people play this game. What is the probability that at least one of the people scores 0 after making the 8 selections?
    (iv) Given that one of the contestants scores -2, what is the probability that the first two balls drawn were the same colour?