

Math SL PROBLEM SET 91

Section A (Skills/Concepts Consolidation)

1. **(F2.6; F2.7) (CI)** $3^{2x} + 3^x - 6 = 0$ can be written as $(3^x + m)(3^x + n)$, where $m, n \in \mathbf{Z}$,
(Cirrito 7.1.5, p208)
- Find the value of m and the value of n .
 - Hence, find the exact solution for the equation $3^{2x} + 3^x - 6 = 0$ and explain why there is only one solution.

2. **(SP5.9) (CA)** A machine fills plastic bottles with mineral water. The volume of mineral water that the machine pours into each bottle follows a normal distribution with a mean of 498 ml and a standard deviation of 3.4 ml. Each can has a maximum capacity of 506 ml. Find the probability that a bottle chosen by random
(Cirrito 17.2, p557)
- has a volume less than 500 ml;
 - has a volume between 500 ml and 506 ml;

3. **(F2.6; F2.7) (CI)** Solve for x : $2^{2x} - 9(2^x) + 8 = 0$.
(Cirrito 7.1.5, p208)

4. **(SP5.7; SP5.8) (CA)** A random number generator generates the numbers 1, 2, 3, 4 or 5. Let X be the discrete random variable “the number generated”. The probabilities for the scores are given in the table below.
(Cirrito 16.1, p533)

x	1	2	3	4	5
$P(X=x)$	0.3	0.2	0.15	0.1	k

- Find k .
- Find $E(X)$
- What is the probability that the number generated is even?

Suppose six numbers are generated.

- What is the probability all the numbers are even?
 - What is the probability three of the numbers are even?
 - What is the probability at least one of the numbers is even?
 - What is the expected number of even numbers?
5. **(F2.6; F2.7) (CI)** Use the substitution $y = 3^x$ to solve the equation $3^{2x+1} - 3^{x+1} + 1 = 3^x$.
(Cirrito 7.1.5, p208)

Math SL PROBLEM SET 91

6. **(SP5.7) (CA)** Five fair coins are thrown onto the floor. Let X be the discrete random variable “number of heads shown”.

(Cirrito 16.1, p533)

- a. Complete the following table:

Number of Heads x	0	1	2	3	4	5
$P(X = x)$						

- b. Verify that the sum of all the probabilities adds to 1.
c. What is the expected number of heads, $E(X)$?

A game is played with the following rules.

- The contestant pays £1 to play the game.
- The five coins are thrown on the floor. If 0 or 1 coins show heads then the contestant receives no money.
- If 2 coins show Heads, then the contestant gets the £1 returned.
- If 3 or 4 coins show Heads, then the contestant gets the £1 returned, and an additional £1.
- If 5 coins show Heads, then the contestant gets the £1 returned, and an additional

£5 Let Y be the discrete random variable “The Net Amount of Money Received”.

- d. (i) Draw a table illustrating the values Y can have, and the probabilities for these values.
(ii) What is the expected amount of money a contestant can receive playing this game?
(iii) Is the game fair? Give a reason for your answer.

7. **(F2.7) (CI)** Solve for x : $x + 1 = \sqrt{6 - 2x}$.

(Cirrito 2.4.4, p57)

8. **(F2.7; CA6.3) (CI)** Given the polynomial $P(w) = w^4 - 5w^2 - 36$:

(Cirrito 20.2, p649)

- a. Find all real solutions to the equation $w^4 - 5w^2 - 36 = 0$.
b. Determine the domain intervals in which $P(w)$ is increasing and decreasing.
c. Find the w -coordinates of all extrema and classify them.
d. Find the w -coordinates of the inflection points.
e. Sketch $P(w)$, labeling the information from the previous questions.