

Math SL PROBLEM SET 90

Section A (Skills/Concepts Consolidation)

1. **(SP5.8) (CI)** The probability of obtaining a head with a certain biased coin is $\frac{2}{3}$. The coin is tossed three times. Find the probability of

(Cirrito 15.1, p506)

- obtaining all heads;
- obtaining at least one tail.

2. **(SP5.7) (CA)** The probability distribution of the discrete random variable X is given by the following table.

(Cirrito 16.1, p533)

x	1	2	3	4	5
$P(X = x)$	0.3	0.08	k	0.15	.25

- Find the value of k .
 - Calculate the expected value of X .
3. **(SP5.8) (CA)** In a group of 10 people the expected number who wear glasses is 2 and the variance is 1.6. Find the probability that

(Cirrito 16.3, p544)

- a person chosen at random from the group wears glasses;
- exactly 6 people in the group wear glasses.

In a different group of 10 people where the probability of a randomly chosen person wearing glasses is higher, the probability that exactly 6 people wear glasses is 0.219.

- Find the probability that a person chosen at random from the group of 10 wears glasses.

4. **(SP5.9) (CA)** The length of a certain skateboard model is advertised to be 81 cm. The actual length, X metres, follows a normal distribution with a mean of 81.04 cm and a standard deviation of 1.2 cm.

(Cirrito 17.2, p557)

- Find:
(i) $P(X < 80)$ (ii) $P(80 < X < 82)$
- Given that the value of the standard deviation does not change, find the mean length necessary to guarantee that only 1% of skateboards have lengths less than 80 cm. Give your answer accurate to four significant figures.

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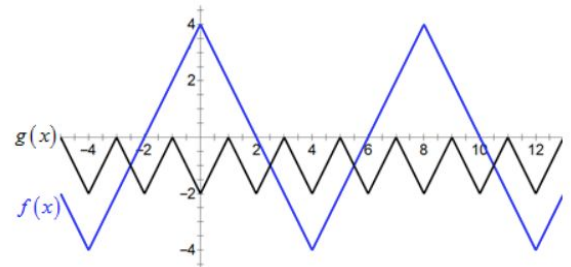
5. **(F2.3; CA6.1; CA6.6) (CI)** Given the function $h(x) = x^2$:
- Show that $g(x) = h(x - 2) - 9 = x^2 - 4x - 5$
 - Describe fully the transformations which map $h(x)$ onto $g(x)$.
 - For the function $y = g(x)$, use the limit definition of a derivative: $\lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$ to derive the equation of the derivative of $g(x)$ from first principles.
 - Evaluate $\int_0^6 |g(x)| dx$

6. **(F2.2; F2.2; CA6.4) (CI)** The function f is defined as $f(x) = \sqrt{4 - x}$.
- State the domain and range of f .
 - Sketch the graph of each function on a separate axis:
 - $y = f(2x)$
 - $y = f(-x)$
 - $y = -\frac{1}{2}f(x) + 2$
 - State the domain and range of $y = f^{-1}(x)$.
 - Find the equation for $y = f^{-1}(x)$ and sketch the graph of f^{-1} .
 - Determine $\int f(x) dx$.

7. **(F2.3; CA6.6) (CI)** The graphs of the functions $f(x)$ and $g(x)$ are shown.

(Oxford 9H, p308)

- Describe fully the transformations which map the graph of $f(x)$ onto the graph of $g(x)$.
- Express $g(x)$ in terms of $f(x)$.
- Evaluate $\int_{-2}^4 f(x) dx$ as well as $\int_{-2}^4 |f(x)| dx$.



8. **(F2.5; CA6.4) (CI)** Let $f(x) = \frac{1}{x-2}$, $x \neq 2$.
- Sketch the graph of f , clearly indicating any asymptotes or any x - or y -intercepts.
 - The graph of function g is obtained by first reflecting the graph of f about the x -axis followed by a translation 3 units in the positive y -direction. Write an expression for the function g and sketch its graph, clearly indicating any asymptotes or any x - or y -intercepts.
 - Determine the equation of the line that is normal to f at the point where $x = 3$.
 - Evaluate $\int_3^7 f(x) dx$.