# Math SL PROBLEM SET 89

**Section A (Skills/Concepts Consolidation)**

1. **(CA6.4) (CI)** Evaluate the following integrals:

**(Oxford 9F, p302)**

a. $\int_0^1 x\sqrt{x^2+3}\,dx$ b. $\int_0^{\pi} \cos x\,\sqrt{\sin x}\,dx$

2. **(T3.4) (CA)** The triangle ABC is such that BC = 10.2, $\angle BAC = x$, $\angle ABC = 2x$ and $\angle ABC$ is an obtuse angle.

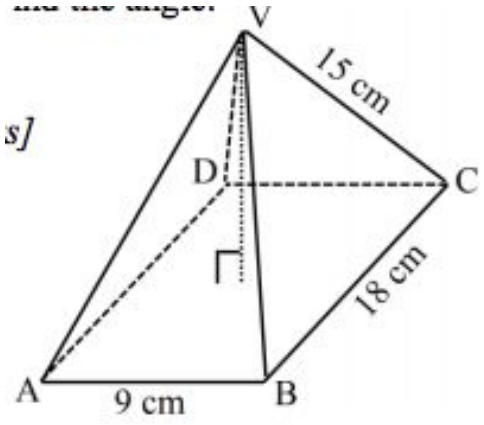
**(Cirrito 9.5.4, p300)**

a. Find AC in terms of $\cos x$.
b. Given that the area of triangle ABC is $52.02\cos x$, find $\angle ACB$.

3. **(T3.4) (CA)** The diagram shows a right pyramid ABCDV with vertex V directly above the centre of the rectangular base ABCD with AV = BV = CV = DV = 15 cm. Find the angle:

**(Cirrito 9.5.3, p285)**

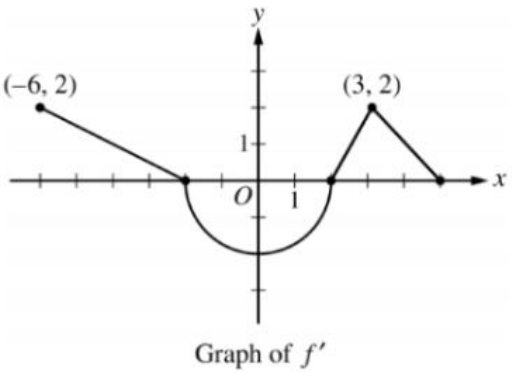
a. between the face ABV and the base;
b. between the edge AV and the base.




4. **(CA6.5) (CI)** The function $f$ is differentiable on $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of $f'$ consists of a semicircle and three line segments, as shown in the figure.

**(Oxford 9H, p308)**

a. Find the values of $f(-6)$ and $f(5)$.
b. On what intervals is $f$ increasing? Justify your answer.
c. Find the absolute minimum value of $f$ on the interval $[-6, 5]$. Justify your answer.
d. Find the value of $f''(-5)$ and $f''(2)$ or explain why it does not exist.



Graph of $f'$

5. **(T3.6) (CA)** Find the possible length(s) of side AC of triangle ABC given that AB = 13.8 cm, BC = 10.7 cm and angle A = 34.7° .

**(Cirrito 9.5.1, p290)**

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**Section B (Skills/Concepts Practice)**

6. **(SP5.9) (CA)** A random variable $X$ is distributed normally with a mean of 15 and a standard deviation of 3.

**(Cirrito 17.2, p557)**

   a. Find $P(12.5 \leq X)$.
   b. Find $P(20.5 \geq X)$.
   c. Let $P(X \leq a) = 0.65$ . Find the value of $a$.
   d. Let $P(b \leq X \leq 20.5) = 0.5$. Find the value of $b$.

7. **(SP5.9) (CA)** $X$ is a random variable that is normally distributed with a mean of 5.

**(Cirrito 17.2, p557)**

   a. Given that $P(X < 4) = 0.1057$ , show that the value of the standard deviation (accurate to three significant figures) is 0.8.
   b. Find $P(|X - 5| < 0.5)$.

8. **(SP5.8) (CA)** A bag contains 5 balls, of which 2 are red and 3 are blue. A box contains 8 balls, of which 6 are red and 2 are blue. A ball is removed at random from the bag without replacement. If the ball is red, then another ball is removed at random from the bag. If the ball is blue, then a ball is removed at random from the box.

**(Cirrito 15.1, p506)**

   a. Calculate

      i. the probability that the first ball removed is red.
      ii. the probability that both balls removed are red.
      iii. the probability that the second ball removed is red.
      iv. the probability that the second ball removed is red given that the first one removed was blue.

   b. Harry uses the above process every morning he goes to school to help him decide to ride to school on his bicycle, or to take the bus. If the two balls are the same colour, he rides to school on his bicycle, otherwise he takes the bus.

      i. What is the probability he rides to school on his bicycle on any given day?
      ii. During a school week consisting of five days what is the probability he rides to school on his bike at least two times?