

Math SL PROBLEM SET 87

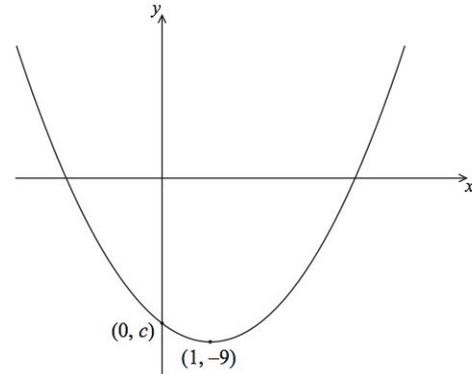
Section B (Skills/Concepts Problems)

1. **(F2.4) (CI)** The following diagram shows part of the graph of a quadratic function f . The vertex is at $(1, -9)$, and the graph crosses the y -axis at the point $(0, c)$. The function can be written in the form $f(x) = (x - h)^2 + k$.

- a. Write down the value of h and of k .
- b. Find the value of c .

Let $g(x) = -(x - 3)^2 + 1$. The graph of g is obtained by a reflection of the graph of f the x -axis, followed by a translation of p .

- c. Find the value of p and q .
- d. Find the x -coordinates of the points of intersection of the graphs of f and g .



2. **(V4.3, V4.4) (CI)** A line passes through the points $A(0, -3, 1)$ and $B(-2, 5, 3)$.

$$\vec{AB} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$$

- a. Show that
- b. Write down a vector equation for L_1 .

$$r = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

A line, L_2 , has equation. The lines L_1 and L_2 intersect at a point C .

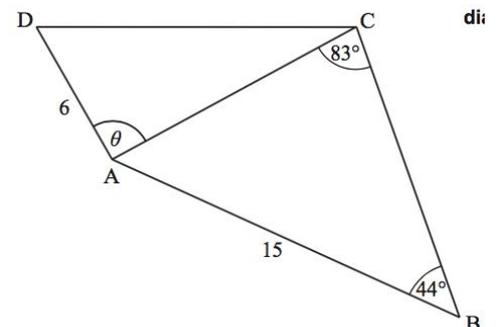
- c. Show that the coordinates of C are $(-1, 1, 2)$.
- d. A point D lies on L_2 so that $|CD| = \sqrt{18}$ and $CA \cdot CD = -9$. Find $\angle ACD$.

3. **(T3.6) (CA)** The following diagram shows the quadrilateral $ABCD$. $AD = 6$ cm, $AB = 15$ cm, $\angle ABC = 44^\circ$, $\angle ACB = 83^\circ$ and $\angle DAC = \theta^\circ$.

- a. Find AC .
- b. Find the area of triangle ABC .

The area of triangle ACD is half the area of triangle ABC .

- c. Find the possible values of θ .
- d. Given that θ is obtuse, find CD .

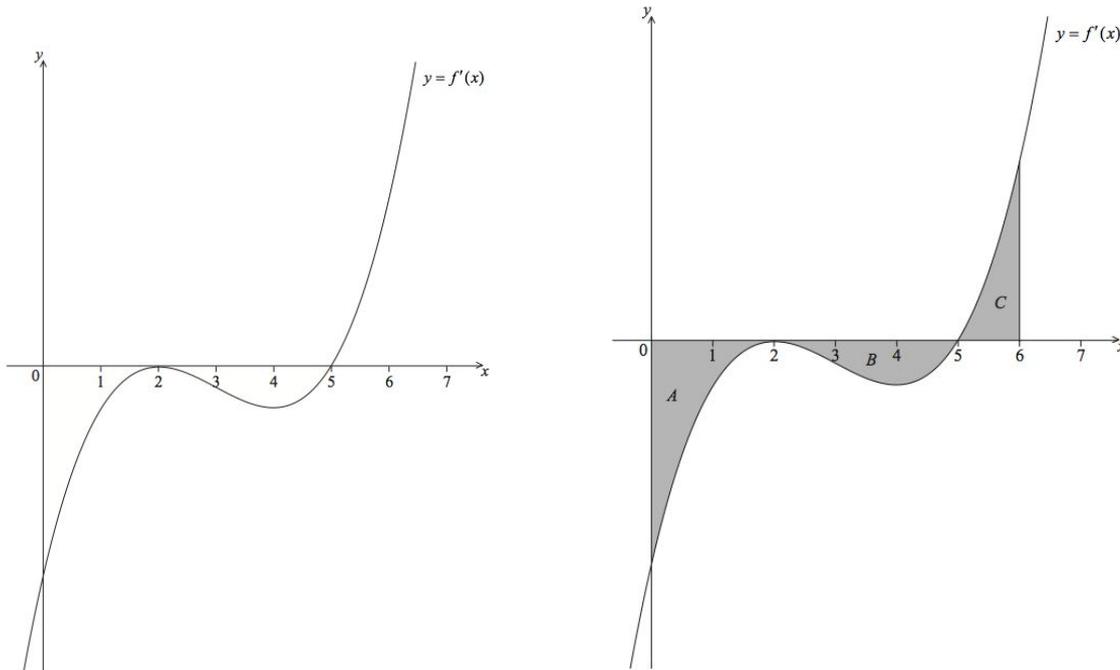


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4. **(C6.3) (CI)** Let $y = f(x)$, for $-0.5 \leq x \leq 6.5$. The following diagram shows the graph of f' , the derivative of f . The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x -axis at the point $(5, 0)$.
- Explain why the graph of f has a local minimum when $x = 5$.
 - Find the set of values of x for which the graph of f is concave down.

The following diagram shows the shaded regions A , B and C . The regions are enclosed by the graph of f' , the x -axis, the y -axis, and the line $x = 6$. The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

- Given that $f(0) = 14$, find $f(6)$.
- Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$.



5. **(SP5.4, F2.6, F2.8) (CA)** An environmental group records the numbers of coyotes and foxes in a wildlife reserve after t years, starting on 1 January 1995. Let c be the number of coyotes in the reserve after t years. The following table shows the number of coyotes after t years. The relationship between the variables can be modelled by the regression equation $c = at + b$.

number of years (t)	0	2	10	15	19
number of coyotes (c)	115	197	265	320	406

- Find the value of a and of b .
- Use the regression equation to estimate the number of coyotes in the reserve when $t = 7$.

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Let f be the number of foxes in the reserve after t years. The number of foxes can be modelled by the equation $f(t) = \frac{2000}{1 + 99e^{-kt}}$, where k is a constant.

- c. Find the number of foxes in the reserve on 1 January 1995.
 - d. After five years, there were 64 foxes in the reserve. Find k .
 - e. During which year were the number of coyotes the same as the number of foxes?
6. The masses of watermelons grown on a farm are normally distributed with a mean of 10 kg. The watermelons are classified as small, medium or large. A watermelon is small if its mass is less than 4 kg. Five percent of the watermelons are classified as small.
- a. Find the standard deviation of the masses of the watermelons.

The following table shows the percentages of small, medium and large watermelons grown on the farm. A watermelon is large if its mass is greater than w kg.

small	medium	large
5%	57%	38%

- b. Find the value of w .

All the medium and large watermelons are delivered to a grocer.

- c. The grocer selects a watermelon at random from **this** delivery. Find the probability that it is medium.
- d. The grocer sells all the medium watermelons for \$1.75 each, and all the large watermelons for \$3.00 each. His costs on this delivery are \$300, and his total profit is \$150. Find the number of watermelons in the delivery.