Math SL PROBLEM SET 82

Section A (Skills/Concepts Consolidation)

1. **(A1.1 - R) (CI)** The first three terms of a sequence are 68, 62, 56.

   a. Write a formula for the n^{th} term, u_n.
   b. If the last term of the sequence is p, how many terms are there in the sequence?
   c. Find a simplified expression for the sum of the n terms of the sequence in terms of p and hence, find the value of p that gives the maximum possible sum.

2. **(T3.6 - R) (CA)** A surveyor measures the angle of elevation of the top of a mountain from a point at sea level as 20°. She travels 1000 meters along a road which slopes uniformly uphill towards the mountain. From this point, which is 100 m above sea level she measures the angle of elevation as 23°. Find the height of the mountain above sea level.

3. **(V4.2, T3.6 - R) (CA)** Point A has the position vector of $-5\boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k}$, point B has the position vector of $6\boldsymbol{i} + 6\boldsymbol{k}$ and point C has the position vector of $8\boldsymbol{i} + 10\boldsymbol{j} + \boldsymbol{k}$. Show that triangle ABC is isosceles and determine the measure of all three angles and the area of the triangle.

4. **(T3.4, C6.2 - R) (CI)** Given the equation $P(x) = \cos 2\left(x - \frac{\pi}{3}\right)$ for $0 \leq x \leq 2\pi$.

   a. Solve $P(x) = 0$ and solve $P(x) = ½$.
   b. Find the solutions to the differential equation $\frac{d}{dx} P(x) = 0$.
   c. Given your work in (a) and (b), sketch a graph of $P(x)$.

5. **(T3.5 - R) (CI)** The temperature A °C inside a house at time t hours after 4:00 am is given by the equation $A = 21 - 3\cos\left(\frac{\pi t}{12}\right)$, $0 \leq t \leq 24$ and the temperature B °C outside the house at the same time is given by $B = 22 - 5\cos\left(\frac{\pi t}{12}\right)$, $0 \leq t \leq 24$.

   a. Find the temperature inside and outside the house at 8:00 am.
   b. Write down an expression for D = A - B, the difference between the inside and outside temperatures.
   c. Sketch the graph of D for $0 \leq t \leq 24$.
   d. Determine when the temperature inside the house is less than the temperature outside.
   e. At what rate is the temperature inside the house changing at 8:00 am?

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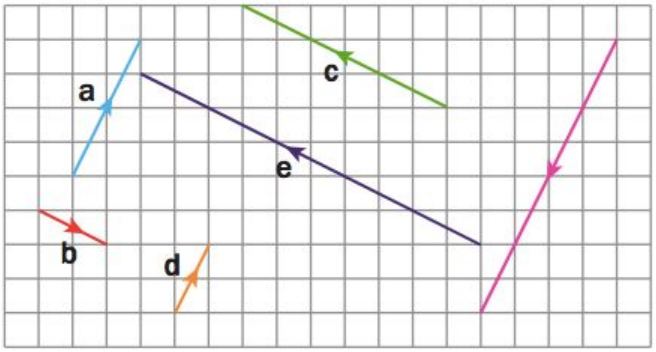
Section B (Skills/Concepts Practice)



6. **(V4.1) (CI)** SKILL: Vector Basics. The diagram shows several vectors.
   a. Write vectors ***a*** and ***b*** in column form and in vector form and find their magnitudes.
   b. Write each of the vectors ***c***, ***d***, ***e***, and ***f*** in terms of vectors ***a*** or ***b*** (as scalar multiples)
   c. How are ***a*** and ***b*** related?

7. **(V4.1) (CA)** SKILL: Vector Basics. Use vectors to show that the points F(3,-4,0), G(-1,8,-8) and H(6,-13, 6) are collinear.

8. **(V4.1) (CA)** SKILL: Vector Basics. Given that $\boldsymbol{a} = x\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k}$ and $|\boldsymbol{a}| = 3x$, find:
   a. two possible values of x.
   b. a unit vector in the same direction as ***a***.

9. **(T3.3) (CI)** SKILL: Trigonometric Identities. If $\sin(\theta) = \frac{2}{7}$ where $\frac{\pi}{2} \leq \theta \leq \pi$, find
   a. $\sin(2\theta)$ b. $\cos(2\theta)$ c. $\tan(2\theta)$ d. $\sin(4\theta)$

10. **(T3.3, T3.5) (CI)** SKILL: Trigonometric Identities. Solve the following for $0 < x < 2\pi$.
    a. $\sin(x) = \sin(2x)$ b. $\sin(x) = \cos(2x)$

Section C (Skills/Concepts HW)

11. Oxford, Chap 12B, p414, Q3,5,6
12. Oxford, Chap 12C, p416, Q2,6
13. Oxford, Chap 12D, p417, Q3,4
14. Oxford, Chap 12E, p418, Q4,5