

Math SL PROBLEM SET 74

Section A (Short Answer)

1. **(F2.6 - R) (CI)** A function is defined by $f(x) = 1 - e^{-x}$.
(Cirrito 7.3, p220)
 - a. Sketch the graph of f and show that an inverse function exists.
 - b. Find the domain of f^{-1} and find the equation for $f^{-1}(x)$.
 - c. Sketch the graph of f^{-1} on the same axes as the graph of f .
 - d. Determine the equation of $(f^{-1} \circ f)(x)$.

2. **(SP5.9 - R) (CA)** A brand of soft drink is sold in “litre” bottles. The amount of liquid in each bottle is a normally distributed random variable with a mean of 1.005 litres and a standard deviation of 0.01 litres.
(Cirrito 17.2, p568)
 - a. Find the proportion of bottles containing less than 1 litre.
 - b. If I buy five bottles, find the probability that at least 2 of them contain less than one litre.
 - c. For a second brand of soft drink, the probability that the bottle contains at most 995 ml of drink is 0.38974, while the probability that the bottle contains more than 1050 mL is 2.74289%. Find the mean and standard deviation of the amount of liquid in the bottles of the second brand.

3. **(SP5.6 - R) (CI)** For events A and B , it is known that $P(A' \cap B') = 0.3$ and that $P(A) = 0.2$ and that $P(B) = 0.6$. Find:
(Cirrito 15.2, p510)
 - a. $P(A \cup B)$
 - b. $P(A | B)$
 - c. $P(B' | A')$
 - d. Are A and B independent events?

4. **(CA6.6 - E) (CA)** The velocity, v , in ms^{-1} of a particle moving in a straight line is given by the function $v(t) = 10\sin\left(\frac{\pi}{16}t\right)$ m/s.
(Cirrito 22.6, p764)
 - a. Given that $s(0) = 0$, find its displacement equation.
 - b. Find its displacement after 20 seconds.
 - c. Find its displacement during the 20th second.
 - d. How far has it travelled in twenty seconds?

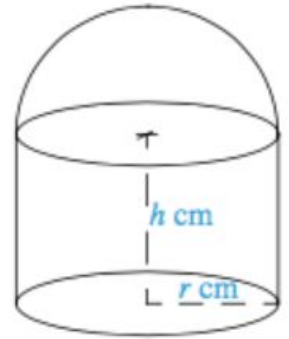
5. **(V4.2 - R) (CA)** Find a unit vector perpendicular to both $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $3\mathbf{j} - 2\mathbf{k}$.
(Cirrito 12.3, p415)

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6. **(F2.1, F2.6 - R) (CA)** A closed tin is to be constructed as shown in the diagram. It is made up of a cylinder of height h cm and a radius base r cm which is surmounted by a hemispherical cap.

(Cirrito 21.4, p716)

- Find an expression in terms of r and h for:
 - its volume, V cm³.
 - its surface area, A cm².
- Given that the volume is πk^3 , $k > 0$, show that its surface area is given by $SA(r) = \frac{2\pi k^3}{r} + \frac{5\pi}{3}r^2$.
- Find the ratio of $r : h$ for A to be a minimum.



Section B (Extended Response/Investigation)

7. **(CA6.5 - N) (CA)** The region a is bounded by the x -axis and the curve $y = \sin(2x)$ between $x = 0$ and $x = \frac{\pi}{2}$. The lines $x = x_1$ and $x = x_2$ and $x = x_3$ cut the region into four parts of equal area.

(Oxford 9.6, p318)

- Find the values of x_1 , x_2 and x_3 .
 - The entire region between $x = 0$ and $x = \frac{\pi}{2}$ is now rotated around the x -axis forming a solid of revolution. Determine its volume.
8. **(CA6.5 - E) (CI)** For the function $h(x) = \sin^2(x) - \cos(x)$, $x \in [0, 2\pi]$, determine:
- all extrema (and express as ordered pairs).
 - Classify the extrema using the second derivative.
 - Hence, determine the intervals of increase and decrease. (Sketch from your answers from Q(a) and Q(b) may help).

(Cirrito 20.2, p649)