

Math SL PROBLEM SET 71

Section A (Short Answer)

1. **(CA6.2 - R) (CI)** Find the equation of the derivatives of the following: **(Cirrito 19.3, p632)**

a. $f(x) = \sqrt{\frac{2x+5}{7x-9}}$ b. $f(x) = \frac{\frac{1}{x} + \frac{1}{x^2}}{x-1}$ c. $y = \tan(\cos(x))$ d. $y = \ln(xe^{7x})$

2. **(V4.4 - R) (CA)** The following pairs of lines may OR may not intersect at a point. Find the point at which the lines intersect OR explain how you know they don't. **(Cirrito 12.7.3, p460)**

a. $L_1: \mathbf{r} = (1,3,5) + \lambda(7,1,-3)$ b. $L_1: \mathbf{r} = (3\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}) + t(-2\mathbf{i} + \mathbf{j} - \mathbf{k})$
 $L_2: \mathbf{r} = (4,6,7) + \mu(-1,0,2)$ $L_2: \mathbf{r} = (5\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) + s(-4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

3. **(SP5.7 - R) (CA)** The discrete random variable x has the probability function given by

$$P(x) = \begin{cases} \left(\frac{1}{4}\right)^{x-1} & x = 2, 3, 4, 5, 6 \\ k & x = 7 \\ 0 & x > 7 \end{cases}$$

(Cirrito 16.2, p541)

- a. Find the value of k .
b. Find $P(X=5 | X < 7)$
c. Find the expected value of x .
4. **(CA6.6 - E) (CI)** The velocity, v , in ms^{-1} of a particle moving in a straight line along the x -axis is given by the function $v(t) = \sin(\pi t)$. **(Cirrito 22.6, p764)**

- a. Determine when the particle is moving to the right and when it is moving to the left and when it is stopped.
b. At any time it stops, determine whether it changes direction or not.
c. Find the particle's displacement for the time interval $0 \leq t \leq 3$.
d. Find the particle's total distance travelled for the time interval $0 \leq t \leq 3$.

5. **(SP5.9 - R) (CA)** The average time it takes high speed trains to travel between Paris and London is 2 hours and 15 minutes with a standard deviation of σ . **(Cirrito 17.2, p557)**

- a. If the probability that the trip takes 2 hours and 10 minutes is 0.1056, show that $\sigma = 4$.
b. What is the probability that a trip will take more than 2 hours and 17 minutes?
c. What is the interquartile range of a trip on these fast trains?

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6. **(SP5.8 - R) (CA)** Habib is a BMW car salesman in Maadi (he didn't quite make it as a test driver!!) He has noticed that 10% of the cars he sells are "metallic grey" in colour. Twenty of his customers are selected at random and their new car orders are checked for colour preferences. Find the probability that: **(Cirrito 16.3, p548)**

- At least 5 cars are metallic grey in colour.
- At most 6 cars are metallic grey in colour.
- More than 5 are metallic grey in colour.

In a sample of 100 customers records, find:

- The expected number of metallic grey car order.
- The standard deviation of metallic grey car orders.

Section B (Extended Response/Investigation)

7. **(CA6.5 - N) (CA)** Find the exact volumes of revolution that are obtained when the following graphs are rotated about the x -axis: **(Oxford 9.6, p318)**

- $f(x) = \sqrt{2x}$ between $x = 0$ and $x = 4$.
- $g(x) = \frac{2}{x}$ between $x = 1$ and $x = 3$.

8. **(V4.3 - R) (CI)** At 12:00 noon, Satellite A is orbiting and is currently above Cairo at a height of 120 km and a speed of 800 km/hr. The current direction vector of satellite A is given as (4,3). NOTE: the vector (1,0) means a displacement of 1 km due east and (0,1) means a displacement of 1 km due north. **(Cirrito 12.7.2, p452)**

- Using Cairo as the origin (0,0), show that the position vector of satellite A at time t hours

after noon is given as
$$r = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 640 \\ 480 \end{pmatrix}$$

- Find the position of the satellite at 14:30 pm.
- Satellite B is heading towards Cairo with a velocity vector of (-300, -400) from a location of (600, 480) and also at a height of 120 km.
 - Find the speed of Satellite B.
 - Is there a danger of collision?