Math SL PROBLEM SET 67

Section A (Short Answer)

1. **(T3.5 - R) (CI)** Solve for θ in the equation $2\sin^2\theta = 3\cos\theta$, where $0 \leq \theta \leq 2\pi$.
(Cirrito 10.4, p351)

2. **(SP5.7; SP5.9 - R,E) (CA)** Answer the following probability distribution questions:
(Oxford 15B, p525; Oxford 15J, p543)

   a. The probability distribution of a discrete random variable X is defined by the following equation: $P(X = x) = cx(6 - x)$, where $x = 1, 2, 3, 4, 5$.
      i. Find the value of c.
      ii. Find $E(X)$.

   b. The mass of packages of washing powder is normally distributed with a mean of 500 g and a standard deviation of 20 g.
      i. Find the probability that a randomly chosen package has a mass more than 475g.
      ii. Three packages are chosen at random. What is the probability that 2 of the 3 have a mass **less than** 475g?

3. **(SP5.3 - R) (CA)** The events G and H are **independent** and it is given that $P(G \cap H') = 0.12$ and $P(G' \cap H) = 0.42$. **(Oxford 3F, p84)**

   a. Draw a Venn diagram to represent the events G and H.
   b. Let $P(G \cap H) = x$. Find the two possible values of x.

4. **(CA6.2, CA6.2 - E) (CI)** Find the second derivative of the function $g(x) = \frac{x^2 - 1}{2x + 3}$ and hence or otherwise, find the inflection point(s) or justify that the function has no inflection points.
(Cirrito 19.3.6, p623)

5. **(V4.2 - R) (CA)** The line L_1 has a vector equation $r = \begin{pmatrix} 2 \\ -3 \\ -3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$. A second line, L_2, is perpendicular to L_1 and is represented by $r = \begin{pmatrix} 3 \\ 12 \\ 5.5 \end{pmatrix} + q\begin{pmatrix} 7 \\ x \\ 1 \end{pmatrix}$. **(Cirrito 12.7, p444)**

   a. Show that $x = -3$.
   b. Find the coordinates of the intersection point of L_2 and L_1.

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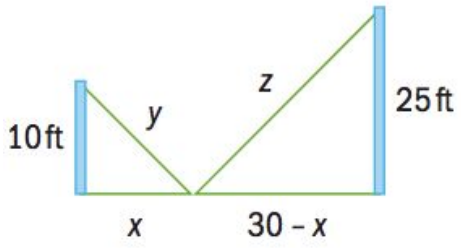
6. **(CA6.4 - E) (CI)** Given that $\frac{d}{dx} f(x) = \sqrt[3]{x} + x^3 + 1$ and that $f(1) = 2$, determine: **(Cirrito 22.4, p740)**
   a. the equation for $f(x)$.
   b. The value of the $\int_1^2 (\sqrt[3]{x} + x^3 + 1)dx$ (You can use a calculator for some of the numerical calculations, but not for evaluating the integral.)

Section B (Extended Response/Investigation)

7. **(CA6.3 - N) (CA)** A 10 foot post and a 25 foot post stand 30 feet apart and are perpendicular to the ground. Wires of lengths y and z run from the top of each pole and are attached by a single stake at a point on the ground between the two poles, as shown in the figure.
(Cirrito 21.4, p716)

   a. Write down an expression for y in terms of x.
   b. Write down an expression for z in terms of x.
   c. Hence, write an expression for L(x), the total length of the wire used for both poles.
   d. Find $\frac{dL}{dx}$.
   e. Hence, or otherwise, find the distance, x, the stake should be placed from the ten foot pole in order to minimize the amount of wire used. (use calculator for this)



8. **(CA6.3 - E) (CA)** The diagram below shows a sketch of the graph of the function $y = \sin(e^x)$ where $-1 < x < 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B. **(Cirrito 20.2, p649)**

   a. Find the coordinates of A.
   b. The coordinates of C may be written as $(\ln k, 0)$. Find the **exact** value of k.
   c. Write down the y-coordinate of B. (Hint: CA??)
   d. Find $\frac{dy}{dx}$.
   e. Hence, show that at B, $x = \ln \frac{\pi}{2}$.
   f. Write down the integral which represents the shaded area.
   g. Evaluate this integral. (Hint: CA???)