

# Math SL PROBLEM SET 66

## Section A (Short Answer)

1. (F2.6 - R) (CI) Solve each of the following equations for  $x$ , giving exact values in terms of natural logs (ln) or in terms of  $e$  (if necessary) **(Cirrito 7.3, p217; Cirrito 7.4, p221)**

a. Solve  $3^x = 6$                       b. Solve  $\ln(3x + 1) - \ln(4 - x) = \ln 4$

2. (V4.2 - R) (CI) The following diagram shows quadrilateral ABCD, with **(Cirrito 12.4, p423)**

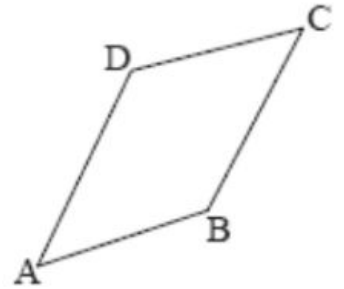
$$\vec{AD} = \vec{BC}, \quad \vec{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad \vec{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

a. Find  $\vec{BC}$

b. Show that  $\vec{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

c. Show that vectors  $\vec{BD}$  and  $\vec{CA}$  are perpendicular.

d. Find the measure of the angle at A.



3. (CA6.2 - R) (CI) Differentiate (and simplify) the following: **(Cirrito 19.3, p618)**

a.  $y = \frac{1}{x^2} \ln(x)$                       (b)  $y = \sqrt{1 + \sin^2(x)}$                       (c)  $y = \frac{\sin(x)}{1 - \cos(x)}$

4. (CA6.2; CA6.5 - R) (CI) Given the function  $f(x) = \left(x + \frac{1}{\sqrt{x}}\right)^2$ , **(Cirrito 19.3.6, p623)**

a. Expand  $f(x)$ .                      b. Differentiate  $f(x)$ .                      c. Evaluate  $\int_1^4 \left(x + \frac{1}{\sqrt{x}}\right)^2 dx$ .

5. (A1.1 - R) (CA) The sum of the first five terms of a geometric series is 3798, and the sum to infinity is 4374. **(Cirrito 8.2.4, p263)**

a. Find the sum of the first seven terms.  
b. Find the value of  $n$  such that  $S_n$  first exceeds 4200.

6. (V4.2 - R) (CA) The position vectors of the points A, B and C are  $i - j + 2k$ ,  $2i + j + 4k$  and  $3i + 4k$  respectively. Find **(Cirrito 12.5, p429)**

a. the angle BAC to the nearest degree;  
b. the area of triangle ABC.

7. (CA6.3 - E) (CI) Given the function  $h(x) = \frac{x-2}{(x-1)^2}$ ,  $x \neq 1$ . **(Cirrito 20.2, p649)**

a. Determine the equation(s) of the horizontal and vertical asymptote(s).  
b. Determine the  $x$ - and  $y$ -intercepts (if possible).

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- c. Find the derivative of  $h(x)$ , writing your answer in the form of  $\frac{a-x}{(x-1)^n}$ , where  $a$  and  $n$  are constants to be determined (by you of course ..... )
- d. Show that the second derivative is  $h''(x) = \frac{2x-8}{(x-1)^4}$ .
- e. Find the coordinates of the extrema(s) and inflection point(s).

## Section B (Extended Response/Investigation)

8. **(SP5.9 - N) (CA)** Recall the formula for the normal distribution. Here are two questions that deal with simple applications of this formula and your graphing calculator (provided you correctly adjust the formula .... and your window settings)

$$f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(X-\mu)^2}{2\sigma^2}}$$

- a. Households in Portugal spend an average of €100 per week on groceries with a standard deviation of €20. Assuming that the distribution of grocery expenditure follows a normal distribution, what is the probability of a household spending:
- Less than €130 per week,
  - More than €90 per week,
  - Between €80 and €125 per week?
- b. A machine produces bolts with diameters distributed normally with a mean of 4 mm and a standard deviation of 0.25 mm. Bolts are measured accurately and any which are smaller than 3.5 mm or bigger than 4.5 mm are rejected. Out of a batch of 500 bolts, how many bolts would be acceptable?
- c. Of course you don't need to memorize this formula (it's not even in the formula book....), so wouldn't it be nice if it was pre-programmed into our calculator!!!
- Go to  $Y =$  on your TI-84 and select  $Y1 =$
  - Go to **2nd DISTR** on your TI-84 and select the 1: normalpdf( command .... So that will now get pasted into your  $Y1 =$
  - Enter the following  $\Rightarrow$  x-value:  $x$  ;  $\mu = 14$  and  $\sigma = 4$  for the next question
  - Now "paste" and then adjust your windows and graph ....
  - Answer the following question: The length of time that SL Math students take to understand how to work with normal distribution curve is known to be normally distributed with a mean of 14 minutes and a standard deviation of 4 minutes.
    - Find the probability that David will take more than 20 minutes to "get this stuff"
    - What percentage of students (like Habib) would be expected to understand "this stuff" in less than 10 minutes?