Math SL PROBLEM SET 65

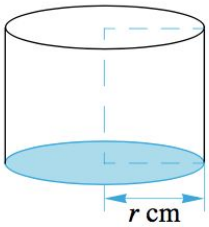
Section A (Short Answer)

1. **(CA6.2 - R) (CI)** Differentiate (and simplify) the following: **(Cirrito 19.3, p618)**

   a. (i) $y = x^2\sin(x)$ (ii) $y = (x^3 - 2x + 1)e^x$
   b. (i) $y = \ln(x + \cos x)$ (ii) $y = (1 - 3x^2)^4$
   c. (i) $y = \frac{x^2 + 1}{sin(x)}$ (ii) $y = \frac{e^x + x}{x + 1}$

2. **(CA6.3 - E) (CI)** Given the function $g(x) = \frac{x}{x^2 + 1}$, determine the x coordinates of the stationary points and inflection points. Predict the end behaviour of the function and prepare a sketch (use your calculator for this if you wish.) **(Cirrito 20.2, p649,; Cirrito 20.3, p672)**

3. **(CA6.3 - E) (CA)** A cylindrical tin with no lid is to be made from a sheet of metal measuring 100 cm^2. **(Cirrito 21.4, p716)**



   a. Given that the radius of the base of the tin is r cm, show that its volume, V cm^3, is given by $V(r) = ½ (100r - \pi r^3)$
   b. Determine the value of r that will give the greatest volume. Use the second derivative test to confirm your value.

4. **(C6.5 - E) (CI)** Given the functions $f(x) = x^2 - 2$ and $g(x) = -x$. **(Cirrito 22.5.8, p755)**

   a. Sketch a graph showing the region bounded by these two functions.
   b. Write down an expression that gives the area of the region.
   c. Hence, or otherwise, find the area of this region.

5. **(V4.3 - R) (CA)** A line, L_1, goes through point P whose position vector is $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and this line is parallel to the vector $3\boldsymbol{i} + 7\boldsymbol{j}$. Find: **(Cirrito 12.7, p444)**

   a. the vector equation of this line;
   b. the parametric equation of this line;
   c. the Cartesian equation of this line;
   d. the angle the line makes with the x-axis

6. **(SP5.9 - N) (CA)** To continue introducing the normal distribution for continuous data,
   a. Watch this video from jbstatistics ⇒ **(Cirrito 17.2, p557)**
   https://www.youtube.com/watch?v=iYiOVISWXS4&t=1s

$$f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(X - \mu)^2}{2\sigma^2}}$$

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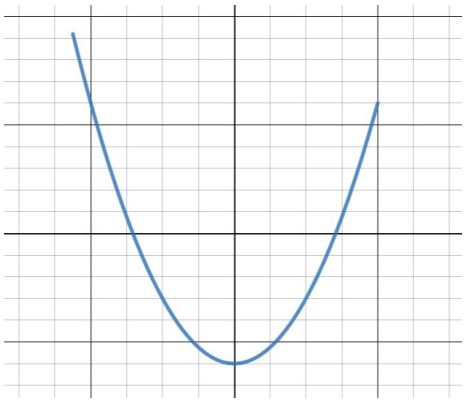
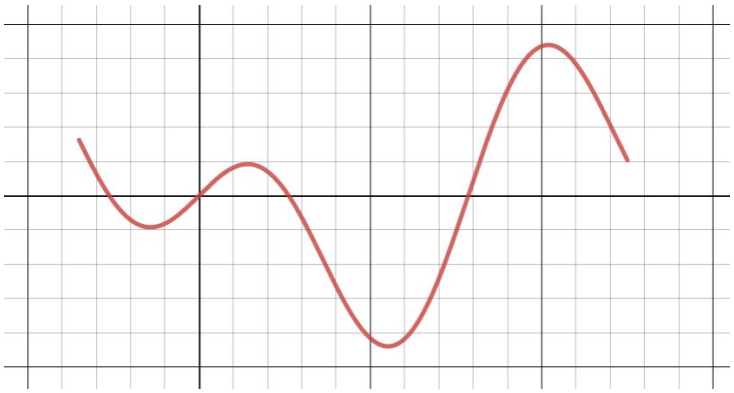
b. Above is the equation for this normal distribution curve, say for student grades on IB SL exams worldwide. Its shape is controlled by two parameters, the mean, μ, and the standard deviation σ. So, graph this normal distribution of student grades on your calculator and substitute in $\mu = 67$ and $\sigma = 10$ into the equation. Draw your graph and state your window settings.

c. Use your graph to determine $P(45 < x < 90)$. Interpret your answer.

Section B (Extended Response/Investigation)

7. **(CA6.3 - R) (CI)** Here is a sketch of two different functions, each of which represent the graph of the **derivative** of some function. For each graph, sketch both the (i) anti-derivative (so in other words, the original function) as well as the derivative of the derivative graph (so in other words the second derivative) **(Cirrito 20.2, p649, Cirrito 20.3, p672)**





8. **(SP5.9 - N) (CA)** To continue introducing the normal distribution for continuous data, use the equation again and investigate the effect of changing the two key parameters. **(Cirrito 17.2, p557)**

$$f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(X-\mu)^2}{2\sigma^2}}$$

a. Set $\mu = 50$ and $\sigma = 10$. Now change σ to 20 and explain/draw what happens.
b. Set $\mu = 50$ and $\sigma = 10$. Now change σ to 5 and explain/draw what happens.
c. Set $\mu = 50$ and $\sigma = 10$. Now change μ to 70 and explain/draw what happens.
d. Set $\mu = 50$ and $\sigma = 10$. Now change μ to 30 and explain/draw what happens.

e. Set $\mu = 50$ and $\sigma = 10$. Use your TI-84 to determine:
   i. $P(20 < x < 80)$
   ii. $P(x < 45)$
   iii. $P(x > 40)$