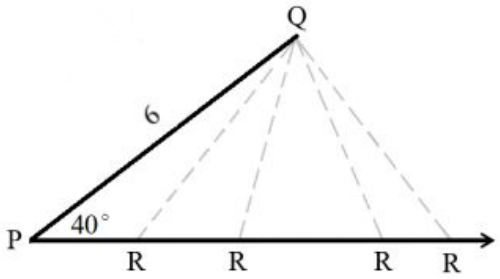
Math SL PROBLEM SET 63

Section A (Short Answer)

1. (**A1.1 - R**) (**CA**) Find all pairs of numbers, a and b, such that 3, a, b are consecutive terms in a geometric sequence, and a, b, 9 are consecutive terms in an arithmetic sequence. **(Cirrito 8.2.3, p261)**

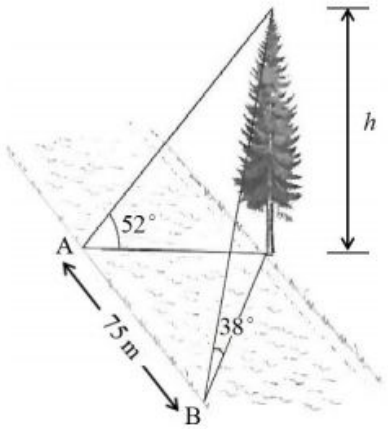
2. (**C6.2 - N**) (**CA**) Consider the function $f(x) = \frac{x^2 + 2x + 1}{x - 1}$.**(Cirrito 5.3.5, p144; 19.3.5, p622)**
   a. Sketch a graph of the function.
   b. Show that $\frac{dy}{dx} = \frac{x^2 - 2x - 3}{(x-1)^2}$.
   c. Determine the domain and range of the function.
   d. Find the equation of the normal to the graph of the function at $x = -\frac{1}{2}$ and express it exactly in the form $y = mx + c$.
   e. Find the coordinates of the point where the normal in (d) intersects the graph of the function a second time.

3. (**T3.6; SP5.4 - R**) (**CA**) As shown in the diagram below, angle QPR equals 40° and PQ = 6. If the length of QR is a randomly chosen real number between 0 and 6, then what is the probability that it is possible to construct two different triangles? **(Cirrito 9.5.2, p294)**



4. (**SP5.5 - R**) (**CA**) In a school there is a group of 32 students who play basketball, volleyball or football. No student plays only volleyball, or only football. The number who play only basketball is the same as the number who play both basketball and volleyball but not football. The number of students playing both volleyball and football but not basketball is four times the number playing all three sports. There is at least one student playing all three sports. Four students play both basketball and football but not volleyball. Given that the number of students playing both volleyball and football but not basketball is less than the number playing both basketball and volleyball but not football, find the number of students who play only basketball. **(Cirrito 15.2, p508)**

5. (**T3.6 - R**) (**CA**) The diagram shows a river with parallel banks (edges). A tree on a bank of the river is **directly opposite** from point A on the opposite bank of the river. Maria wishes to compute the height of the tree. From point A she measures the angle of elevation to the top of the tree to be 52°. She then walks 75 metres along the bank to point B where she measures the angle of elevation to the top of the tree to be 38°. Determine the height of the tree (in metres) accurate to three significant figures. **(Cirrito 9.6, p307)**

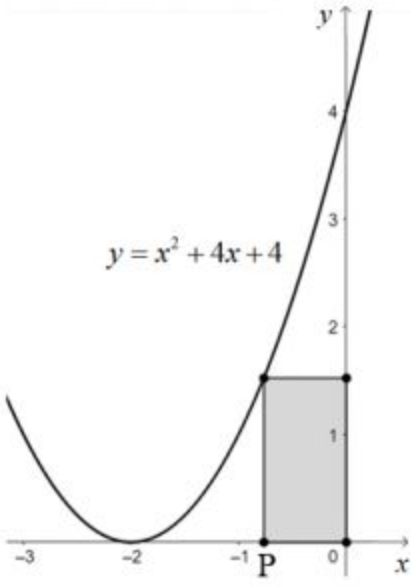


Math SL PROBLEM SET 63

6. (**SP5.2 - R**) (**CI**) There exists a set of five positive integers that has a mean of 5, a median of 5, and a single mode of 8. Determine the value of each of the five numbers. **(Cirrito 13.3, p474)**

Section B (Extended Response/Investigation)

7. (**C6.3 - N**) (**CA**) A rectangle is inscribed in the region bounded by the parabola $y = x^2 + 4x + 4$, the x-axis and the y-axis, as shown in the diagram. One vertex of the rectangle is on the parabola, one is on the y-axis, one is at the origin, and one (labelled point P) is on the x-axis. The coordinates of P are $(p, 0)$ such that $-2 < p < 0$. Find the exact value of p so that the area of the rectangle is a **maximum**. Also, compute the exact maximum area. **(Cirrito 21.4, p702)**



8. (**C6.3, 6.6 - N**) (**CA**) A particle moves in a straight line and passes a fixed point B with a velocity of v_B cm s^{-1} . The particle's acceleration, a cm s^{-2}, is given by $a(t) = 3t^2 - 16t + 12$ for $0 \leq t \leq 6$, where t is the time in seconds after passing point B. It is known that the velocity of the particle is 13 cm s^{-1} when $t = 1$. **(Cirrito 21.3, p694)**

   a. Find the velocity equation for the particle; namely find v_B.
   b. During the interval from $t = 0$ to $t = 6$ seconds, determine:
      i. the number of times the particle changed direction;
      ii. the particle's maximum velocity.
   c. At $t = 6$ seconds, how far is the particle from point B?
   d. Find the total distance the particle travelled during the interval $0 \leq t \leq 6$.