Math SL PROBLEM SET 59

Section A (Short Answer)

1. (**F2.4 - R**) (**CI**) Given the function $f(x) = x^2 - 2x - 8$, determine the: ***(Cirrito 2.4.2, p44)***

   a. values of $f(3)$ and $f(5)$,
   b. average rate of change between $f(3)$ and $f(5)$,
   c. vertex, d. the zeroes, e. the range, if the domain were $-4 \le x \le 5$.

2. **(SP5.5 - R)** (**CA**) A box contains 7 blue marbles and 5 yellow marbles. Mark takes out three marbles, one after another, without replacement. Determine **(Oxford 3.5, p89)**

   a. the probability that Mark takes out three blue marbles;
   b. the probability that Mark takes out exactly 2 yellow marbles.
   c. Given that Mark has at least one blue marble, find the probability that Mark has taken out exactly 2 yellow marbles.

3. **(F2.6 - E)** (**CA**) Use **Symbolab** to simplify the following expressions. Comment on any generalizations that you may be observing. **(Cirrito 7.4, p225)**

   a. (i) $5^{log_5 x}$ (ii) $5^{log_5 x^2}$ (iii) $5^{3log_5 x}$ (iv) $25^{log_5 x}$
   b. (i) e^{lnx} (ii) e^{ln2x} (iii) e^{2lnx} (iv) e^{xlna}

4. (**F2.1, 2.3 - R**) (**CI**) Given: $f(x) = e^{2x}$ for all x and $g(x) = \frac{3}{2}\ln(x)$ for $x > 0$. ***(Cirrito 5.4.1, p148 & p157)***

   a. State the ranges of both $f(x)$ and $g(x)$.
   b. Explain why both functions have inverse functions.
   c. Find expressions for both $f^{-1}(x)$ and $g^{-1}(x)$.
   d. Solve the equation $fog(x) = gof(x)$.

5. (**F2.2, 2.5 - R**) (**CI**) Given the function $f(x) = 2 + \frac{1}{2x-5}$, $x \neq \frac{5}{2}$; ***(Cirrito 5.3.5, p144)***

   a. Write down the equation of each of the asymptotes,
   b. Determine the value of each of the intercepts,
   c. Sketch the curve of f for $-3 \le x \le 5$, showing the asymptotes and intercepts.

6. **(T3.6 - R)** (**CA**) Mr. S is about to go zip lining! He notices that the angle of depression of the zip line is 14°. If the starting platform is 100 m high and the finishing platform is on the ground, **(Cirrito 9.5, p290)**

   a. how long is the zip line?
   b. how much "ground distance" is there between the starting platform and the finishing platform?
   c. You had to make an assumption to answer Q(a) and Q(b). Now assume that this assumption was NOT true and the ground "sloped away" at an angle of 6°. Now re-determine the (i) length of the zip line and (ii) the ground distance between the two platforms.

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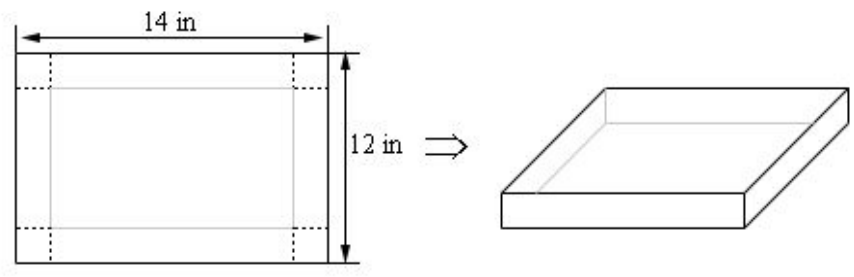
Section B (Extended Response/Investigation)

7. **(C6.3 - N)** (**CA**) We have a piece of cardboard that is 14 inches by 12 inches and we're going to cut out the corners as shown below and fold up the sides to form a box, also shown below. You will determine the height of the box that will give the maximum volume. **(Cirrito 21.4, p702)**

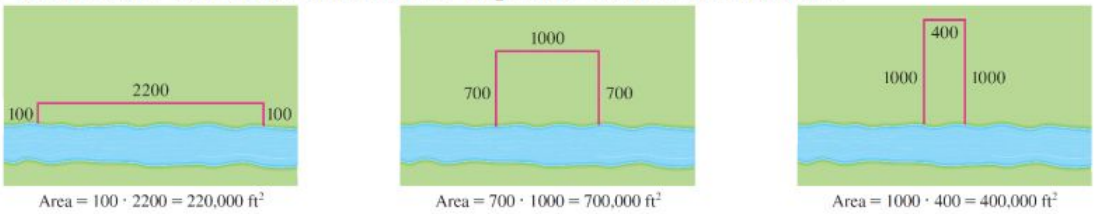


   a. Show that the volume formula for the box is $V(h) = 168h - 52h^2 + 4h^3$, where h is the resultant height of the box.
   b. Find the value of h that optimizes the volume of the box.
   c. Use the **second derivative test** to verify that your value for h does give a **maximum** volume.

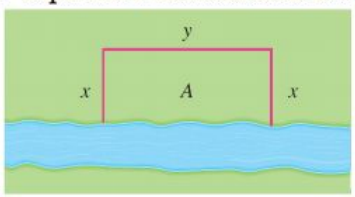
8. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. You will determine the dimensions of the field that has the largest area.

(Cirrito 21.4, p702)

Solution: Note that the area of the field depends on its dimensions:



To solve the problem, we first draw a picture that illustrates the general case:



   a. Show that the area equation for the field is $A(w) = 2400w - 2w^2$, where w represents the width of the field.
   b. Find the value of w that optimizes the area of the field.
   c. Use the **first derivative test** to verify that your value for w does give a **maximum** area.